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THESIS

REFINEMENT AND EXTENSION OF SHRINKAGE TECHNIQUES IN LOSS RATE ESTIMATION OF MARINE CORPS OFFICER MANPOWER MODELS

by

Charles R. Dickinson

March 1988

Thesis Advisor: R. R. Read

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This thesis is a continuation of previous work to apply modern multiparameter estimation techniques to the problem of estimating attrition rates for a large number of small inventory cells in manpower planning models used by the U.S. Marine Corps. The main advances involve the promising introduction of empirical Bayes (non-constant shrinkage) techniques, recognition of the non symmetric nature of the errors with a response to this, and some insight into all aggregation plans that should help provide greater stability for the estimation methods. In addition, the roles of some middle level methodological choices are explored.

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Refinement and Extension of Shrinkage Techniques in Loss Rate Estimation of Marine Corps Officer Manpower Models

by

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Captain, United States Marine Corps
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Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

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March 1988

ABSTRACT

This thesis is a continuation of previous work to apply modern multiparameter estimation techniques to the problem of estimating attrition rates for a large number of small inventory cells in manpower planning models used by the U.S. Marine Corps. The main advances involve the promising introduction of empirical Bayes (non-constant shrinkage) techniques, recognition of the non symmetric nature of the errors with a response to this, and some insight into all aggregation plans that should help provide greater stability for the estimation methods. In addition, the roles of some middle level methodological choices are explored.

563

THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

TABLE OF CONTENTS

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For my wife, Dorie, who basically functioned as a single parent during the preparation of this thesis. By freeing me to work as many hours as needed, you made this effort as painless as possible. Your contributions did not go unnoticed.

I. INTRODUCTION

A. PURPOSE

In support of project OPUS (Officer Planning and Utility System), Headquarters, U.S. Marine Corps, in 1985, requested assistance in exploring new methods of generating manpower loss rates in order to improve upon the currently used method of force rate analysis. In response, students and faculty in the Operations Research department at the Naval Postgraduate School began exploring the use of modern multiparameter estimation techniques for this problem. Special emphasis is placed on the *small cell* problem, i.e., categories of officer skill, grade and length of service which have low inventory figures. Historically, rate estimators for small cells are unstable and a large number of these cells exist.

This paper builds upon previous work on this problem. The main advances are to append new measures of effectiveness as requested by the Navy Personnel Research and Development Center (NPRDC), introduce a class of empirical Bayes estimators, and explore the effects of some middle level choices in applying the new and existing methods. The ultimate goal is to refine the techniques presented here in order to validate a clear policy for predicting loss rates.

B. BACKGROUND

For an introduction into Marine Corps policy concerning manpower planning, the reader is referred to a thesis submitted in September 1985, at the Naval Postgraduate School, by Major D.D. Tucker [Ref. 1], who presents a detailed background into the Marine Corps officer structure and the manpower planning process. Tucker also provides basic attrition rate theory and calculates overall attrition rates in several different formats as they pertain to the Marine Corps. As the aggregation rate begins to grow smaller with refinement, Tucker illustrates the irregular behavior of losses due to voluntary attrition and the small numbers of losses obtained in such aggregates. Tucker [Ref. 1:p. 50] introduces the James-Stein technique of loss rate estimation as a *shrinking* of individual cell averages toward a grand average in order to reduce the risk, or squared differences of forecast and actual values, the goal being an improvement over the classical maximum likelihood estimator, referred to from now on as the MLE. Tucker chose the ranks of First Lieutenant and Lieutenant Colonel for evaluation using military occupational specialty (MOS) groups of combat support and ground combat (three MOS's were selected for each group). These aggregation schemes were carried through other studies and are included for historical purposes in this paper.

Another thesis submitted at the Naval Postgraduate School in March 1986, by Major J.R. Robinson [Ref. 2] focused on a technique called limited translation of the James-Stein estimator. This particular model attempts to minimize the risk mentioned above by reducing the shrinkage of rate estimates for cells which fall outside a certain range of values centered on the grand mean. It was thought that shrinking all cells by the same rate toward this grand mean may be unwise since the attrition rates for those particular cells, those farthest from the grand mean, may contain attributes which are different than the majority of the cells in the aggregate. Additionally, Robinson's work carries forward the study done by Tucker concerning MLE and James-Stein estimators.

These feasibility studies were limited in scope due to the format of the summary date tape available (acquired from NPRDC), which allows only coarse cell definition. A cell is defined to be a cross-classification of the forty military occupational fields (OF), thirty-one lengths of service (LOS), and ten grades for a total of 12,400 categories for manpower planning purposes [Ref. 2:p. 10]. The data does not distinguish between limited duty officers, those officers specifically designated for limited duty within certain MOS's, and unrestricted officers (regular and reserve) [Ref. 1:p. 20]. These factors dictated a broad aggregation scheme that created stable cell inventories and allowed the common variance assumption crucial to their work. But some of the application models require more refined information along those lines and experiments with finer aggregation levels, necessary in some real cases, caused this assumption to be violated.

Tucker and Robinson did conclude that the current scheme of attrition rate prediction could be improved upon by their methods, although no dominant scheme was uncovered. Tucker and Robinson did identify problem areas in their studies that required additional work. Tucker [Ref. 1:p. 71] cited a need for a better aggregation method to produce a more homogeneous attrition behavior in all cells. Also, small probabilities of loss within a cell were not dealt with successfully. Robinson [Ref. 2:p. 32] revealed the small cell problem and the inability to normalize the cell means or stabilize the variance with a data transformation. The recommendations of the previous studies highlighted the need for a more refined data tape with current information to include, among others, full Military Occupational Specialty (MOS) information, grade separation to include regular/reserve status, promotion zone data and breakdown of attritions by type.

C. PROGRESS

The project of loss rate estimation, sponsored by NPRDC, is currently moving into the implementation stage. An operational data tape is now available which includes detailed information on Marine Corps officers for the years 1977-86. Work has begun at the Naval Postgraduate School to break out the data in a useful format and its availability in now nearing completion.

In addition to the results stated earlier, Tucker and Robinson provided loss rate estimates for small cells with no attrition, i.e., MLE equal to zero, and risk effects of the different schemes studied. These results provided the springboard for this study and the requirements for additional future studies.

Captain R.W. Larsen [Ref. 3] has developed a promising aggregation scheme based on cluster analysis, i.e., a classification scheme to aggregate cells which reflect a greater degree of homogeneity in attrition rates than are allowed using current aggregation methods. Table 1 displays the resulting scheme, given in year of current service (YCS) groups proposed for implementation by Larsen.

Table 1. AGGREGATION METHOD PROPOSED BY LARSEN

MOS Category	Bounded YCS Groups			
Fixed-Wing Pilots	(1-6,8-19)	(7)	(20-25)	(26)
Rotary-Wing Pilots	(1-5,8-19)	(6,7)	(20-25)	(26)
Naval Flight Officers	(1-5,8-19)	(6,7)	(20-25)	(26)
Lawyers	(1-6,8-19)	(7)	(20-25)	(26)
All Else	(1-3,6-19)	(4,5)	(20-25)	(26)

The parentheses encompass YCS groups which behave similarly. In each case the grade is fixed, i.e., one should aggregate over YCS before aggregating over grade.

This information will allow for more successful application of the loss rate estimation techniques of interest here when the new data tape is provided.

D. GOALS

In an attempt to refine the methods of attrition rate estimation presented in the previous pilot studies, the following goals were set for this paper:

1. Modify the existing methods to:
   a) use the refined data format,
   b) extend the work to the case of unequal or non constant variance in the small cell inventory problem,
   c) study alternate transformation inversion techniques,
   d) introduce and evaluate additional measures of effectiveness suggested by NPRDC to include, but not limited to, cross validation underages and overages of the deviation between forecasts and actuals,
   e) examine the graphical effect of shrinkage in the original scale.

2. Introduce empirical Bayes estimation method in several forms for attrition rates for consideration as another option to solve the problem.

II. EXTENSIONS TO ESTIMATION METHODS

A. GENERAL

As stated in Chapter I, Tucker and Robinson published pilot studies introducing the James-Stein, limited translation James-Stein, Maximum Likelihood and Transform Scale Cell Average estimators as alternatives to the aggregate methods previously used for manpower planning in the Marine Corps. Their performance was promising, and this chapter will investigate ways to enhance their performance and possibly develop a dominant estimator for future attrition prediction. Additionally, NPRDC has expressed interest in additional measures of effectiveness for these estimators which may lead to a sharper direction for the goal of producing a viable estimation policy. This chapter concentrates on the following issues concerning previous studies.

B. NON CONSTANT VARIANCE

For a cell having an inventory n, and attrition rate p, the number of attritions y is described by the Binomial (n,p) distribution. The variance of the estimator of $\hat{p} = y/n$ is given by the familiar formula

$$\mathrm{var}(\hat{p}) = \frac{p(1-p)}{n} \qquad (2.1)$$

The Freeman-Tukey double arcsine transformation was used by Tucker [Ref. 1:p. 55] and Robinson [Ref. 2:pp. 74-75] in order to map the raw losses to the transformed scale. This formula is given by

$$x = 0.5[n + 0.5]^{1/2}\{\sin^{-1}[\frac{2y}{(n + 1)} - 1] + \sin^{-1}[\frac{2(y + 1)}{(n + 1)} - 1]\} \qquad (2.2)$$

Note that n=n(t) may change with time and thus, so would x. The question of averaging over time before or after applying the Freeman-Tukey transformation is not discussed in this paper. One of the goals of using this transformation is to stabilize the variance of this estimate. In the work of Tucker and Robinson, the assumption of normally distributed random variables with common variance was the setting in which the James-Stein estimator was expected to perform well. Indeed, for n moderate or large and p not too extreme, the variance of the Freeman-Tukey transformation is approximately one. But as discovered by Robinson, this assumption does not hold well when applied to cells with small inventory. A more careful look at the variance of the transformed data is taken here, with particular interest in the region of unstable variance where n and p are small.

Some exploratory graphical work has revealed that the natural log of the variance of x, the transformed value, behaves quite well as a linear function of $\mu(=E[X])$ and $\mu-1$ in this unstable zone. This leads to the interpolatory formula:

$$\operatorname{var}(x) = 1.6835\mu^{-.8934}(\mu - 1)^{.9881}, \qquad (2.3)$$

$$1.001 \le \mu \le 2.2$$

The upper limit, $\mu = 2.2$, is the value for which $\operatorname{var}(x) = 1$. Since the least squares fitting process was applied to the logarithms, μ can not be allowed to fall to one or below. The lower limit, $\mu = 1.001$, is an arbitrary value that meets this constraint. The coefficients were validated by a linear regression on the natural log of this formula. The actual variance function for n = 1,2,3,4,5,7,10 appear in Figure 1, and the fit is remarkably good for $n \ge 3$. The upper graph shows the region of special interest for this paper and this portion of the curve is fitted by the formula in Equation 2.3 above. The lower plot exhibits the outer tale effect yet to be explored. The small values of $n(<3)$ do not maintain the stable variance region, i.e., $\operatorname{var}[x]=1$, for long before falling back. As the value for n increases, the more stable the variance becomes. Table 2 is a partial display of the residuals for selected n values, fifteen equally spaced values in the range $1.001 \le \mu \le 2.2$, computed by the linear regression formula above. For values of $n > 2$ the difference between actual and fitted values are considered acceptable for this work.

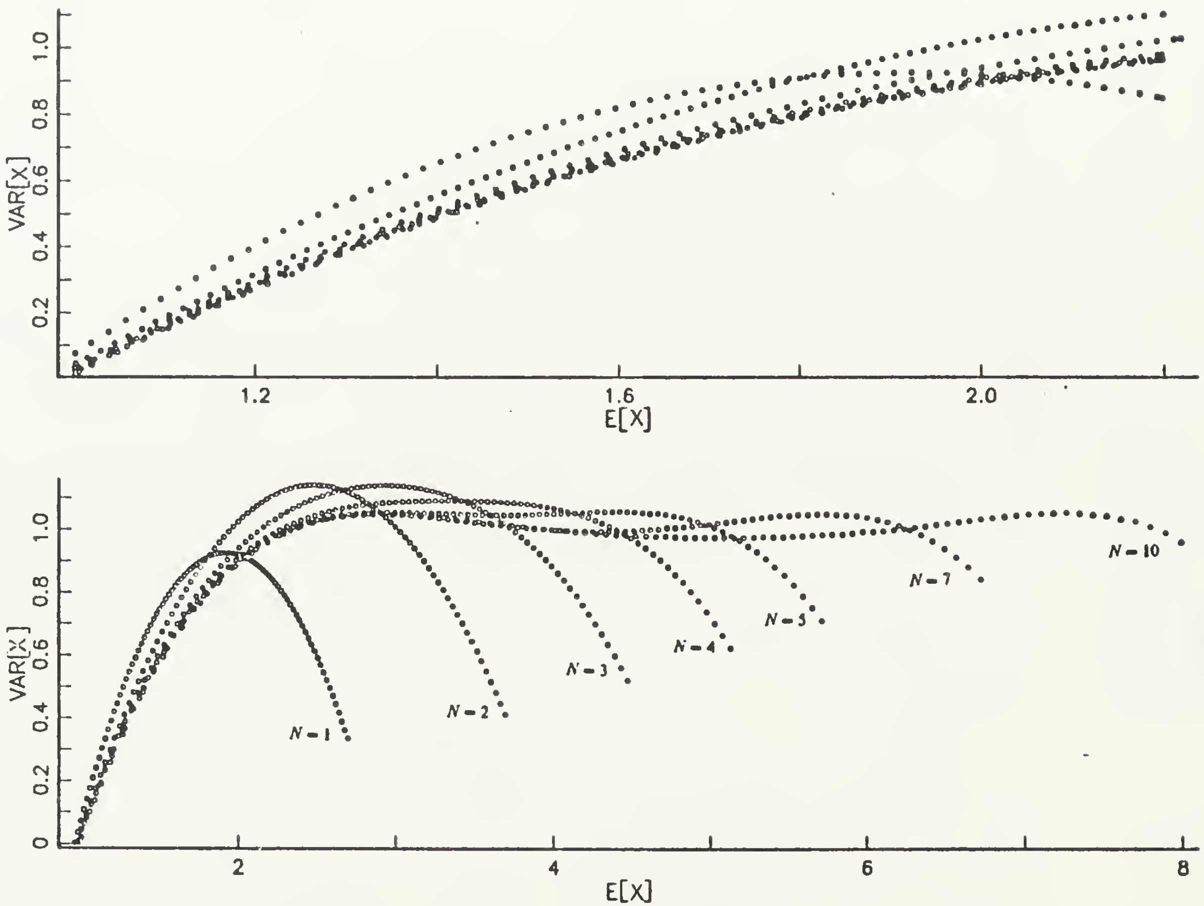

Figure 1. Variance of Freeman-Tukey Transform as Function of It's Mean for Selected n.

Table 2. RESIDUAL LISTING FOR SELECTED VALUES OF N

N=1	N=2	N=4	N=5	N=10
0.07191	0.03912	0.02139	0.017	0.008313
0.08466	0.02569	0.002853	-0.001695	-0.01106
0.1033	0.0239	-0.005448	-0.01049	-0.0206
0.122	0.03002	-0.00655	-0.01185	-0.02257
0.1385	0.04068	-0.004026	-0.009566	-0.02067
0.15	0.05388	0.0000676	-0.005727	-0.0166
0.1552	0.06853	0.004027	-0.002103	-0.01291
0.1527	0.08234	0.007447	0.0009541	-0.009981
0.1414	0.09481	0.00948	0.002557	-0.008299
0.1209	0.1055	0.009825	0.002448	-0.008128
0.08958	0.1129	0.008369	0.0004583	-0.009929
0.04832	0.1169	0.004997	-0.003511	-0.01327
-0.005254	0.1171	0.0001116	-0.00893	-0.0182
-0.06914	0.113	-0.006798	-0.01654	-0.02463
-0.1466	0.104	-0.01534	-0.02585	-0.03366

The boundaries for μ, in the unstable region, as a function of n and p are provided in Figure 2. The formula derived was quite adequate for this study.

C. ALTERNATE TRANSFORM INVERSION METHODS

There are three issues in the inversion of the data from the transformed space. The first is that of selecting values of n, i.e., an average of the n(t)'s appearing in the estimation year inventories. This issue is discussed later in the chapter, with the development of a simulation for comparing mean inventory values. The second is that of the inversion of the sum of the two arcsine functions. The third involves the case of time averaging in the transformed scale and the question of what matching form of average

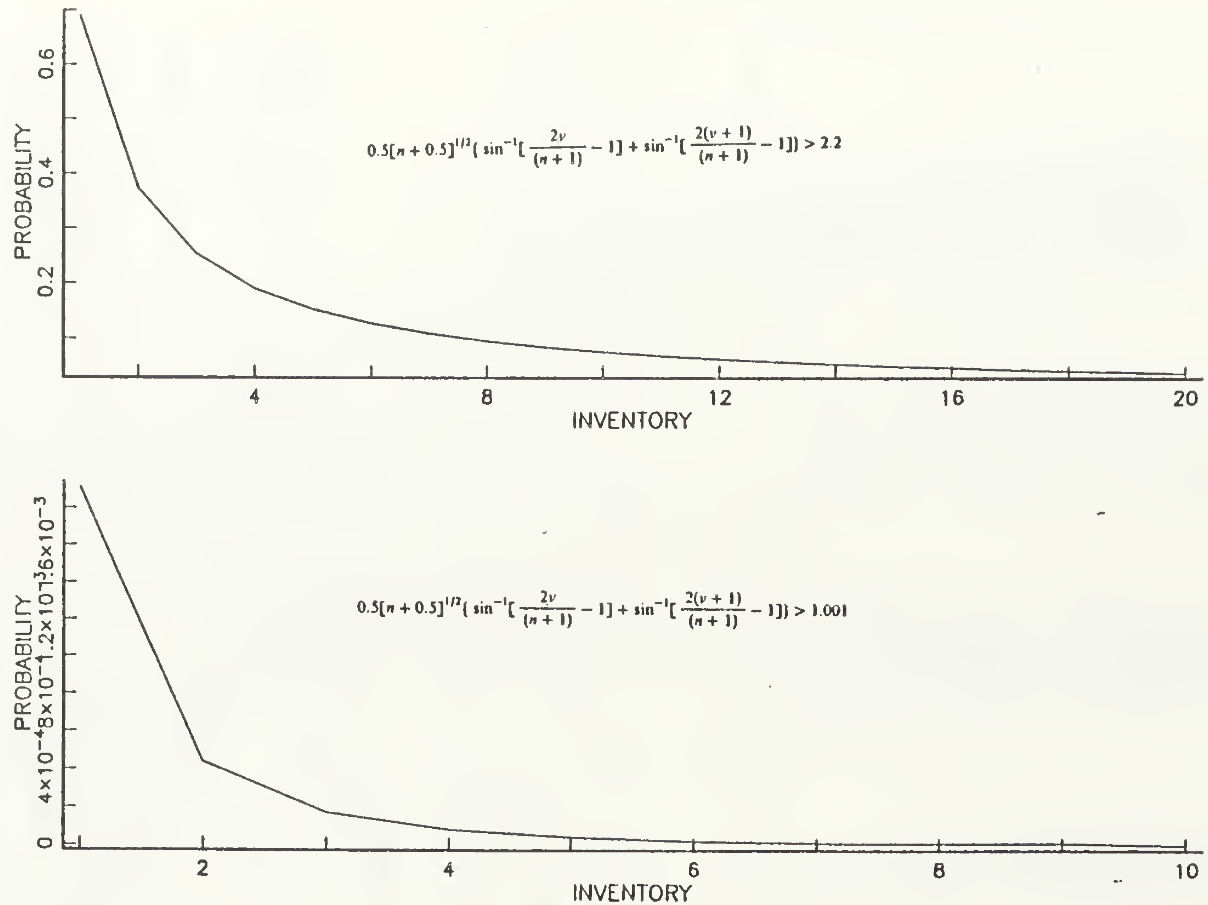

Figure 2. Envelope for the Non-Constant Portion of the Fitted Variance as Functions of Attrition Rate and Inventory.

inventory should be used in the inversion. The third issue is not covered in this paper, and is still pending. Treatment of the second issue will be provided first.

The method used in the pilot studies of Tucker and Robinson, and the most transparent transform inversion formula is:

$$\hat{p}_i = 0.5[1 - \sin(\frac{x}{(n + 1/2)^{1/2}})] \qquad (2.4)$$

This will be referred to as the Basic Inversion. The formula offered by Carter and Rolph [Ref. 4] is given by:

$$\hat{p}_i = \sin^2(\hat{\Theta}_i) + \{\frac{(1 - 2\sin^2\hat{\Theta}_i)}{(4(n/k) + 2)}\}\hat{B}_i \qquad (2.5)$$

where $\hat{\Theta}_i$ is $x/(n + 1/2)^{1/2}$ and $\hat{B}_i$ is the estimated amount of shrinkage in the transformed scale depending on the Bayes model used (constant or proportional prior), k is the number of cells and n is inventory.

Apparently, Carter and Rolph invert prior to shrinking and the above formula provides a shrinkage in the original scale. Rewriting the above, it appears to shrink towards $\hat{p} = 1/2$, and is given by:

$$\hat{p}_i = 1/2(\frac{2\hat{B}_i}{4(n/k) + 2}) + \sin^2(\hat{\Theta}_i)(1 - \frac{2\hat{B}_i}{4(n/k) + 2}) \qquad (2.6)$$

An exact Freeman-Tukey inversion formula developed by Miller [Ref. 5] was recently uncovered. It is exact in the sense that for a single cell, the formula given by:

$$\hat{p}(t) = 0.5\{1 - \mathrm{sgn}(\cos t)[1 - (\sin t + \frac{\sin 5 - 1/\sin t}{n})^2]^{1/2}\} \qquad (2.7)$$

where $t = x/(n + 1/2)^{1/2} + \pi/2$ and a is the average central inventory, returns the empirical rate y/n. This will be referred to as the Freeman-Tukey Exact (FTE) inversion. Since this formula is applied using an average value of n, there may be concern that it oscillates between successive integer values of n. Figure 3 indicates, using values of n=5 and 10, that it is quite smooth as an interpolation formula. In the article, Miller suggested using the harmonic mean of the n values provided by the time changes. This issue is examined later in this chapter. The Miller formula was incorporated in the Marine Corps Inventory Attrition Analysis Program provided in Appendix B.

D. SHRINKAGE VISUALIZATION

When shrinkage is a fixed amount for all cells, it is readily visualized, for all degrees of shrinkage, by the linear diagrams presented in the works of Tucker [Ref. 1:p. 52] and Casella [Ref. 6]. Robinson [Ref. 2:p. 20] also presented diagrams of the limited translation option and how the shrinkage is affected. All of these diagrams refer to the transformed scale.

Of more immediate interest is the question of how to visualize shrinkage diagrams of this type in the original scale. Illustrations of diagrams of this type are provided

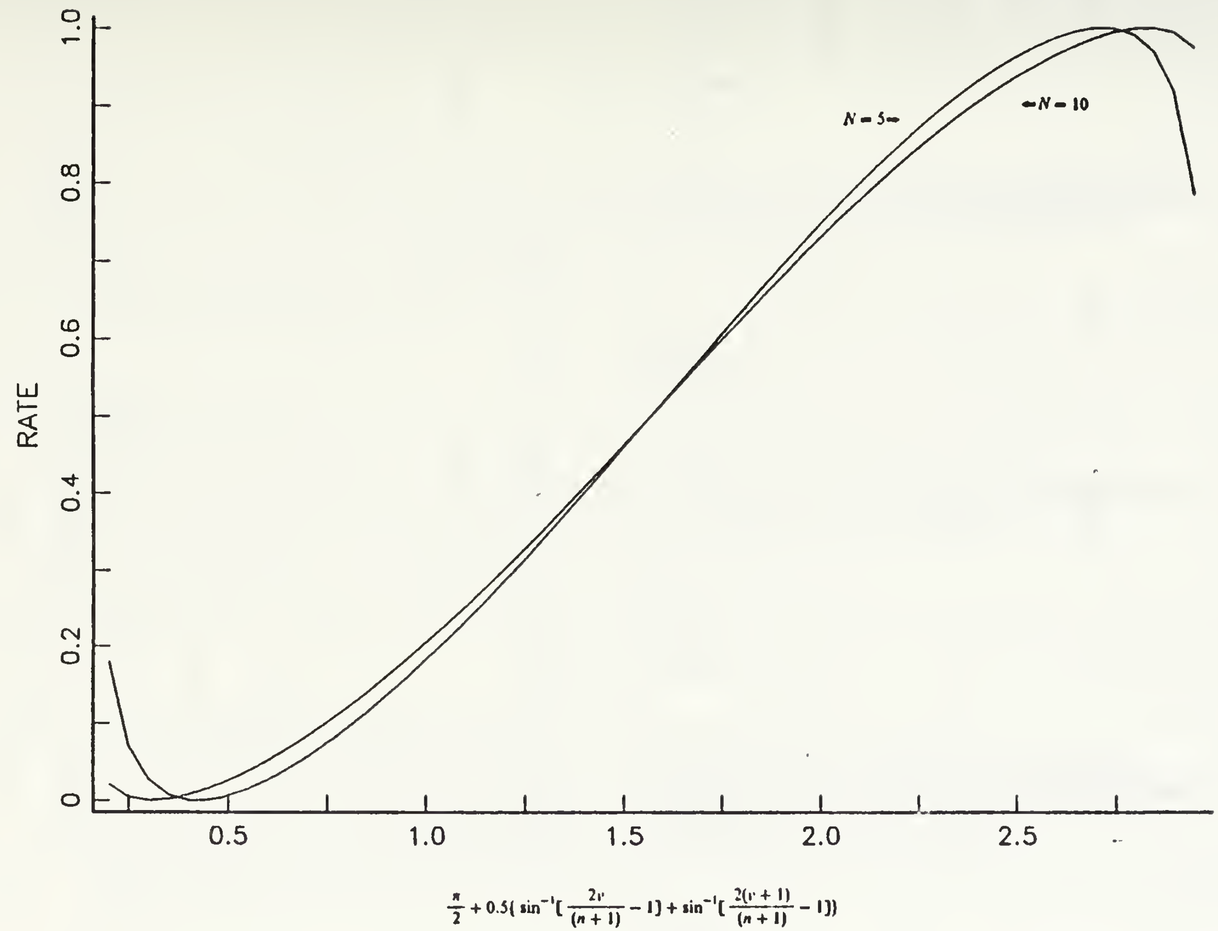

Figure 3. Suitability of the Exact Freeman-Tukey Inversion as an Interpolatory Formula.

using both the basic inversion formula used by both Tucker and Robinson, and the FTE formula. Figure 4 shows how the diagram by Casella [Ref. 6:p. 84] looks in the original scale using the same value of n for all cells. An interesting note to consider is the effect of the difference between inversion formulas, i.e., in the graph which combines the Basic and FTE inverses, the FTE inversion tends to shrink at a slower rate, as values get further from the mean. For a modest transition into the case of non constant n, Figure 5 provides for two values of n. Finally, Figure 6 illustrates the point with many different values of n, taken from actual Marine Corps officer inventory data. In this case, note that Basic inversion shrinks to one original scale value while the FTE inverse is not as focused.

E. CHOICE OF AVERAGE INVENTORY VALUES OVER TIME

A Fortran simulation was developed, included as Appendix C, to test the performance of each of several common means (arithmetic, geometric and harmonic) in the comparison of the inverted estimates with the empirical y/n. Values of n were produced from a Poisson distribution with rate λ ($1 \le \lambda \le 20$), and attrition values, y, were produced from the Binomial (n,p) distribution where a was in the range $.01 \le p \ge .40$. The n and y values were then transformed by using the Freeman-Tukey formula. An exhaustive study of inversion variations with both the FTE formula and the basic formula

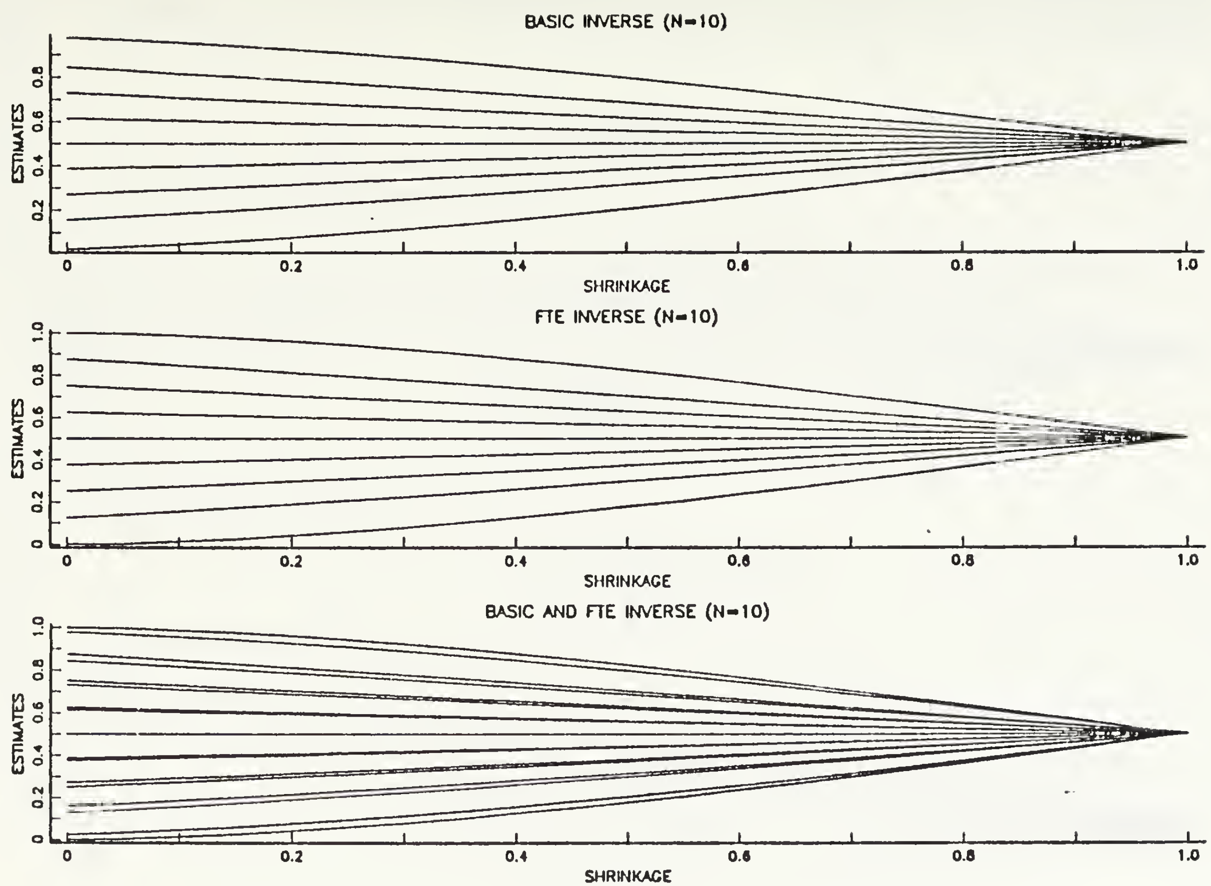

Figure 4. Non Linear Shrinkage Diagrams for a Single Value of Inventory.

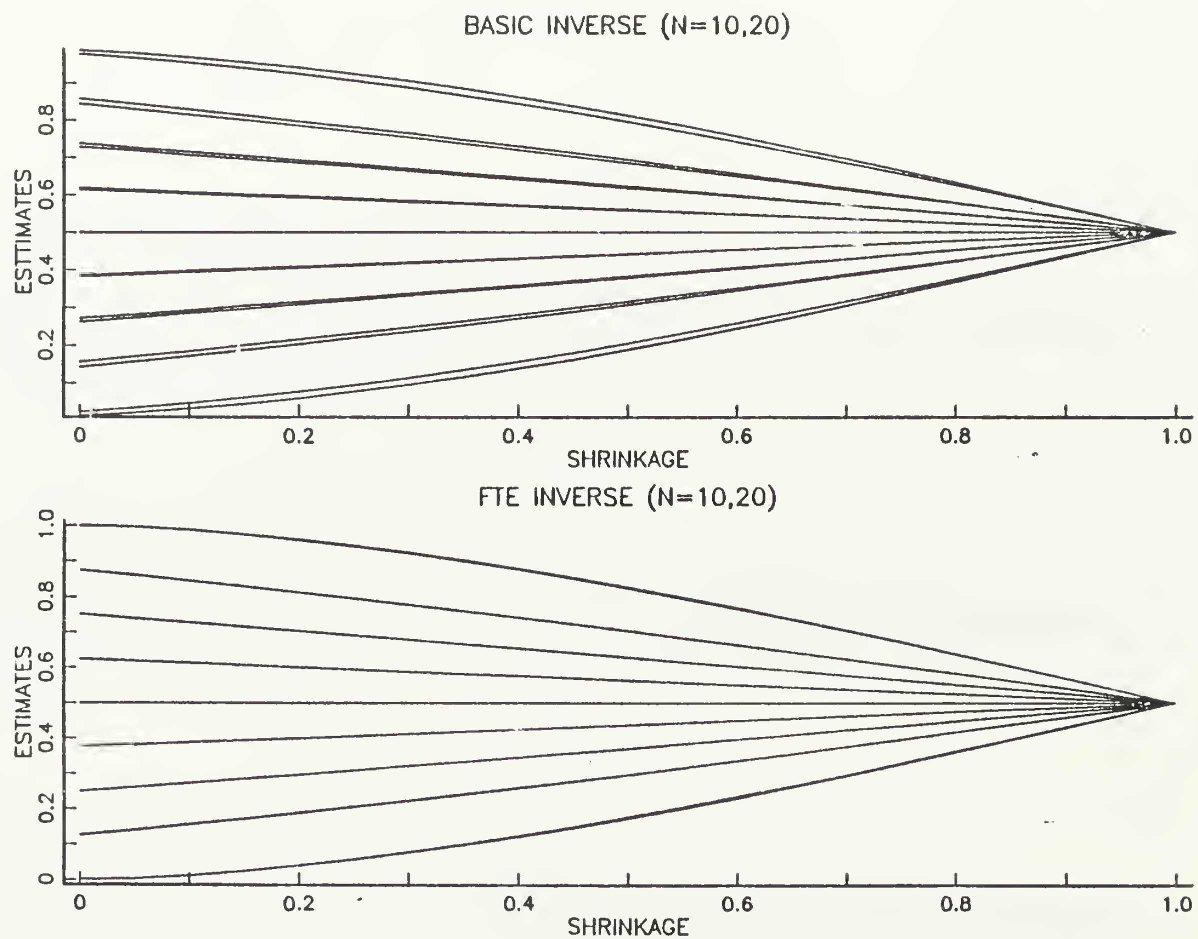

Figure 5. Non Linear Shrinkage Diagrams for Two Values of Inventory.

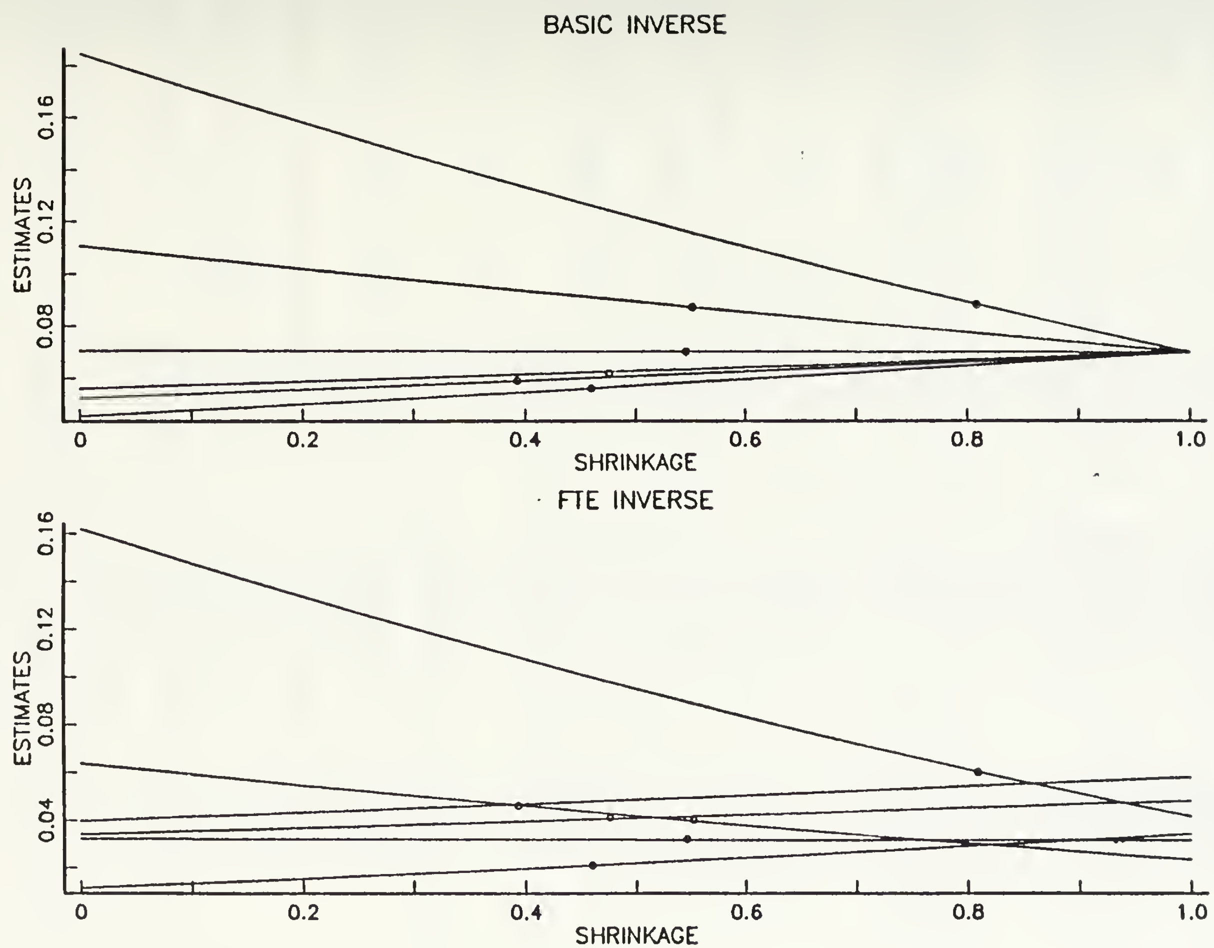

Figure 6. Non Linear Shrinkage Diagrams with Empirical Bayes Estimators for Test Case.

used by Robinson were compared. Table 3 contains selected outputs which are typical of the broader study and shows how the means performed against each other in three different computations listed below.

1. The BASIC Method utilizes the Basic inversion formula and the average transformed scale value.

2. The FTE1 Method utilizes Equation 2.7 with the t values equal to the average (over replications, i.e., time) of $x/(n + 1/2)^{1/2}$ and x given by Equation 2.2. More explicitly, see Equation A.16.

3. The FTE2 Method utilize Equation 2.7 with the t values equal to the average (over replication, i.e., time) of x directly, and x given by Equation 2.2 followed with a division by $(n + 1/2)^{1/2}$

In the context of this study, the arithmetic mean tended to perform as well or better than the geometric or harmonic means. The choice of arithmetic mean also circumvents the problem of an undefined term when $n_i = 0$ is encountered while using the harmonic of geometric means.

F. QUESTIONS RELATING TO VALIDATION

The pilot studies by Tucker and Robinson both used four years (1977-80) for estimation of the attrition rates and the remaining three years (1981-83) for validation. The estimation rates were used in an attempt to forecast the values for the three validation years. The quality of the forecasts deteriorates as lead time increases in the future. This was anticipated, in fact, Rowe et al., at NPRDC have shown a time series effect in their work forecasting attritions in the Navy [Ref. 7] and are currently

Table 3. SIMULATION OUTPUT OF MEAN VALUES FOR INVERSION UTILIZATION

λ	REPS	P	METHOD	ARITHMETIC	GEOMETRIC	HARMONIC
2.0	40	0.05	BASIC	0.1068	0.1176	0.1308
			FTE1	0.0252	0.0189	0.0122
			FTE2	0.0181	0.0208	0.0245
2.0	40	0.10	BASIC	0.1399	0.1537	0.1708
			FTE1	0.0535	0.0458	0.0369
			FTE2	0.0489	0.0558	0.0648
2.0	40	0.20	BASIC	0.2402	0.2629	0.2906
			FTE1	0.1900	0.1829	0.1739
			FTE2	0.1657	0.1869	0.2144
2.0	40	0.40	BASIC	0.3713	0.4038	0.4429
			FTE1	0.3442	0.3403	0.3353
			FTE2	0.3325	0.3715	0.4213
4.0	40	0.05	BASIC	0.0750	0.0815	0.0921
			FTE1	0.0235	0.0200	0.0147
			FTE2	0.0200	0.0221	0.0258
4.0	40	0.10	BASIC	0.1280	0.1387	0.1564
			FTE1	0.0768	0.0724	0.0655
			FTE2	0.0726	0.0798	0.0922
4.0	40	0.20	BASIC	0.1965	0.2125	0.2386
			FTE1	0.1446	0.1406	0.1339
			FTE2	0.1485	0.1628	0.1869
4.0	40	0.40	BASIC	0.4382	0.4694	0.5185
			FTE1	0.4564	0.4558	0.4549
			FTE2	0.4280	0.4638	0.5223
8.0	40	0.05	BASIC	0.0552	0.0580	0.0616
			FTE1	0.0181	0.0166	0.0148
			FTE2	0.0156	0.0165	0.0177
8.0	40	0.10	BASIC	0.1226	0.1285	0.1363
			FTE1	0.0922	0.0904	0.0880
			FTE2	0.0828	0.0873	0.0933
8.0	40	0.20	BASIC	0.2035	0.2131	0.2255
			FTE1	0.1778	0.1762	0.1741
			FTE2	0.1708	0.1797	0.1915
8.0	40	0.40	BASIC	0.3799	0.3962	0.4170
			FTE1	0.3700	0.3693	0.3684
			FTE2	0.3662	0.3837	0.4065

recommending a second order auto regression scheme for this purpose. In this regard, the work of Tucker and Robinson with validation may be viewed as *forecasting by persistence*, and in light of the NPRDC work, can hardly be expected to perform well in an absolute sense. Their work does, however, provide some comparisons that can be useful in choosing among alternative techniques, and much can be learned from them.

The first measure considered is the Mean Squared Error (MSE) of forecasts. The shrinkage estimation theory provides that these values should be about unity if persistence, independence over time, was agreeable. That is, useful validations can be obtained without modeling the time dependence of the attrition process. Only for a very few cases does this hold (i.e., Warrant Officers, see Tucker). However, if one method of estimation consistently produces noticeably smaller MSE values than do the other methods, this would provide sufficient support for that method's continued development.

The second measure is designed to consider performance in the original scale and is patterned after a chi-square statistic

$$\sum_{1}^{k} \frac{(\text{actuals} - \text{forecasts})^2}{\text{forecasts}} \qquad (2.8)$$

If the k cells are independent, then equation 2.8 is approximately a chi-square random variable with k degrees of freedom. Hence, the man value is about k and the standard deviation is $(2k)^{1/2}$. Once again, we generally do not see values this small, but use it for comparison purposes. As a side note, the value of k when using the MLE is generally smaller than that of the other estimators due to the omission of those cells with MLE equal to zero over all estimation years, i.e., zero forecasts. This provides and unnatural advantage for the MLE, which must be kept in mind when interpreting the results.

Two disturbing features have emerged from the exploratory analysis. Often a technique that performs well for some cases using the MSE measure is not comparably supported by the chi-square measure. It seems the two measures do not weight the common features equally, and further study is necessary to develop understanding of the results.

Discussion with personnel at NPRDC concerning these matters has led to the introduction of a third measure, which will be called the Mean Absolute Deviation (MAD). This value is computed in the original scale. Moreover, since the cost structure of over-forecasting is not symmetric, we separate the two parts into the average *overage* deviation given by:

$$\text{ODEV} = \frac{1}{k} \sum_{1}^{k} [\text{forecasts} - \text{actuals}]^{+} \qquad (2.9)$$

and average *underage* deviation

$$UDEV = \frac{1}{k} \sum_{1}^{k} [\text{actuals} - \text{forecasts}]^{+} \qquad (2.10)$$

where the "+" refers to the positive part, as computed separately. Finally,

$$MAD = ODEV + UDEV \qquad (2.11)$$

This measure also has shown that in most cases, the overage exceeds the underage value, which in turn may lead to forecasting the need for an excess number of Marine officers.

III. EMPIRICAL BAYES ESTIMATION

A. BACKGROUND

Although the studies of Tucker and Robinson showed improvement over the current methods, no clear procedure was established and no plans for implementation were drafted. Problem areas were identified while working with small cell inventories and low loss rates. In the transformed scale, the James-Stein assumption of normal distribution of the observations with equal variance did not hold well in such cases. An alternate method of loss rate estimation is required to deal with this special case.

B. INTRODUCTION

Empirical Bayes is simply a generalization of the James-Stein estimator for use with small cell inventories and unstable variance. Random variables are assumed to be of the form:

$$X_i \sim N(\Theta_i, d_i), \; i=1,2,\ldots,k \qquad (3.1)$$

where Θ_i are the unknown parameters to be estimated and d_i are known variances. The Bayesian assumption is that the Θ_i's are themselves from some distribution, in this case also from the normal distribution with mean, ν, and variance, $\rho_i a$. The proportionality constant ρ_i is normalized so that

$$\Sigma \rho_i = k, \; i=1,2,\ldots,k \tag{3.2}$$

where k is the number of observations or cells. Thus,

$$\Theta_i \sim N(\nu, \rho_i a), \; i=1,2,\ldots,k. \tag{3.3}$$

This distribution is known as the *prior* distribution of Θ_i and represents the additional assumed information on the value of Θ before observing X whose distribution depends on Θ [Ref. 8:p. 553]. The use of this information produces an estimate which is a weighted average of the prior mean, ν, and the sample estimate x. In the empirical Bayes context, the values of ν and a, are also estimated from the data since all the information necessary is contained in the marginal distribution of X_i (unconditional on Θ_i) [Ref. 6:p. 83]. This chapter focuses on that method.

For a more detailed explanation of the Bayesian family of estimators, the reader is referred to Casella [Ref. 6], Carter and Rolph [Ref. 4] and Efron and Morris [Ref. 9].

C. THE ESTIMATION METHOD

Fay and Herriot [Ref. 10] discuss the empirical Bayes technique and provide an estimate for the prior mean as

$$\nu^* = \frac{\sum_i x_i/(a^* + d_i)}{\sum_i 1/(a^* + d_i)}, \; i=1,2,\ldots,k \tag{3.4}$$

where a^* is an unbiased estimate of a. The empirical Bayes estimate of Θ, commonly called the mean of the *posterior* where a^* is an unbiased estimate of a. The empirical Bayes estimate of Θ, commonly called the mean of the *posterior* distribution becomes

$$\delta^{EB} = \left(\frac{a^*}{a^* + d_i}\right)x_i + \left(\frac{d_i}{(a^* + d_i)}\right)\nu^*, \quad i=1,2,\ldots,k \qquad (3.5)$$

This is the mean of the conditional distribution of Θ given x [Ref. 8:p. 556]. The weights of each of the prior and sample means are determined by the prior and observed variances.

As a measure of the worth of this estimator, when the number of x's is ≥ 4 [Ref. 11], the *risk*, or sum of squared differences between the true unknown parameter and the empirical Bayes estimate, is less than that using the observed x's, for all Θ_i. Efron and Morris [Ref. 9] provide a rigorous proof.

An important result of the empirical Bayes estimator is that shrinkage values are no longer constant, but depend upon information contained in each cell. Additionally, the point toward which the cell values are shrunk is the weighted mean, not the grand mean of the observations. Appendix A provides the derivation of the empirical Bayes estimator, variable shrinkage factors and calculation of the prior variance used in this study as they pertain to the Marine Corps officer attrition rates.

IV. VALIDATION PROCEDURES AND RESULTS

A. GENERAL

The outputs displayed in this chapter are excerpts from that produced by the Marine Corps Personnel Inventory Attrition Analysis (MCIAAP) (see Appendix B). Initially, comparisons will be made with the output displayed by Robinson to identify the validity of the Fortran code with the APL code used by Robinson, then new approaches will be displayed in an attempt to improve the results. The Fortran code in the MCIAAP incorporates most of the procedures studied by Tucker and Robinson. Specifically, the MLE, Transformed Scale Cell Average (TSCA) and the James-Stein (JS) estimators, shrinkage procedures and a basic inversion formula. The FTE inversion formula, the new measures of effectiveness from NPRDC, are also included. For the MAD measure of effectiveness, only maximum likelihood and the modified empirical Bayes variables will be presented, since these showed the most promise over all. All other variables are displayed by the MCIAAP if interest is warranted by the reader. Chapter II describes all of these methods in some detail.

B. HISTORICAL COMPARISONS

Tables 4-9 were developed using the same aggregation scheme and format as Robinson [Ref. 2:pp. 39-41], although not all of the variables are included (MLE, TSCA, and James Stein are the only variables duplicated in this paper). Additionally, Robinson presented the number of cells, as k values, used for evaluation of each estimate in the aggregate Figure of Merit (FOM) display. These k values are not repeated in the current FOM tables, but are included, as parenthetical values, and discussed in the new MAD measure of effectiveness tables. Since this is the case here, the reader is reminded that the apparent competitive performance of the MLE in the following cases is due, in part, to the possible advantage of having a smaller number of cells used in this MOE's computation.

Since the computations were identical for both studies for these tables, the output was expected to be nearly identical or some pattern discernable between the two. In very few cases was this found to be true, and after verifying that the algorithms coded both in the APL and Fortran versions were correct, we have no reasonable explanation for the discrepancies of the programs. It is suspected that the transfer of raw data into an APL workspace, accomplished by Tucker and Robinson, may be at variance with that into Fortran files, done by contractor.

1. Aviation Aggregate Case

In the case of aviation FOM's, displayed in Table 4, 1981 values for LtCol's in the transformed scale are identical to Robinson's [Ref. 2:p. 39] while the last two

Table 4. AVIATION FIGURES OF MERIT

	1981	1982	1983
	TRANSFORMED FOM		
1st Lt			
MLE	4.0863	10.6590	11.1338
TSCA	3.7420	10.3636	10.8711
JS	3.9694	10.5671	11.1626
MOD JS	4.7352	11.2615	12.1206
EB	4.0753	10.7015	11.3303
MOD EB	3.1833	5.0568	6.2870
Lt Col			
MLE	4.3664	8.5033	8.1506
TSCA	5.7768	10.2994	10.3353
JS	5.7369	10.2468	10.2972
MOD JS	5.7208	10.1865	10.2756
EB	5.8482	10.2298	10.3534
MOD EB	4.3301	3.9824	2.7186
	ORIGINAL FOM		
1st Lt			
MLE	21.0501	43.1546	53.0538
TSCA	56.4603	93.7810	98.4715
TSCAM	53.7884	90.5164	95.3329
JS	58.0150	95.2150	100.1664
JSM	55.3844	91.9406	97.0163
MOD JS	63.0345	99.8419	105.4897
MOD JSM	60.6413	96.5615	102.3172
EB	58.8824	96.2449	101.2482
EBM	56.2111	92.9699	98.0958
MOD EB	19.7683	38.4637	49.7393
MOD EBM	20.7987	36.3673	48.1659
Lt Col			
MLE	33.9315	59.4200	22.1156
TSCA	38.4478	57.0987	69.0641
TSCAM	35.0390	48.9374	59.6957
JS	37.8248	55.1913	67.1696
JSM	33.2016	46.9901	56.7135
MOD JS	38.0291	51.5325	63.3908
MOD JSM	34.5466	45.1554	52.8285
EB	40.3421	54.9389	67.7091
EBM	36.7498	46.9657	57.8567
MOD EB	40.2056	70.5381	60.5749
MOD EBM	185.7476	65.1553	168.3976

years are smaller. The 1stLt FOM's are larger for all three years. The modified empirical Bayes (Mod EB) values were best overall yet still may be too large (ideal values tend to be close to unity). It is interesting to note that in most cases, the risk values tend to increase with future forecasts, yet the Mod EB FOM decreases as future forecasts increase with time.

In the original scale, MLE tended to be smaller than Robinson's for all years, while TSCA and James-Stein (JS) were all larger. The FTE inversion worked best for all variables except for Mod EB in the first and third validation years. MLE and Mod EB worked best for 1stLt's and are approximately within the range of one standard deviation of the number of cells in the computation. For LtCol's, FTE inverted JS was best with MLE a competitive second, yet all were not in the desired range.

In the MAD measure, Table 5, results are mixed between future forecasts (no set pattern) and variables for the two ranks (FTE Mod EB was best for 1stLt's and MLE was best LtCol's). The mix in overage and underage is most desirable in the first year for all cases, yet the number of cells (k) forecasted as overage exceed the underage forecast cell numbers in most cases. This is cause for concern and the performance of these estimates in this aggregate is suspect. The second year values for LtCol estimates displays a preferred ratio of k values, but the total MOE value may still be too large.

Table 5. MEAN ABSOLUTE DEVIATION (AVIATION)

	1981	1982	1983
1st Lt			
MLE			
UNDERAGE	0.1256(3)	0.0439(1)	0.3069(1)
OVERAGE	2.3985(8)	3.9267(10)	4.2522(10)
TOTAL	2.5240(11)	3.9706(11)	4.5591(11)
MOD EB			
UNDERAGE	0.1822(4)	0.0410(1)	0.3281(1)
OVERAGE	2.3625(8)	3.9040(11)	4.2351(11)
TOTAL	2.5447(12)	3.9451(12)	4.5633(12)
MOD EBM			
UNDERAGE	0.2518(5)	0.0580(2)	0.3420(1)
OVERAGE	2.1654(7)	3.6427(10)	3.9677(11)
TOTAL	2.4172(12)	3.7007(12)	4.3098(12)
Lt Col			
MLE			
UNDERAGE	0.6005(5)	1.5139(10)	0.0396(1)
OVERAGE	0.7925(8)	0.3928(3)	1.3467(12)
TOTAL	1.3930(13)	1.9067(13)	1.3863(13)
MOD EB			
UNDERAGE	0.6053(6)	1.4717(9)	0.0841(2)
OVERAGE	1.3083(10)	1.0970(8)	2.2331(14)
TOTAL	1.9136(16)	2.5687(17)	2.3173(16)
MOD EBM			
UNDERAGE	0.6890(6)	1.5366(10)	0.1167(2)
OVERAGE	1.0120(10)	0.7179(7)	1.7501(14)
TOTAL	1.7010(16)	2.2546(17)	1.8669(16)

2. Combat Support Aggregate Case

Table 6 displays the combat support aggregate output from the program. In the transformed scale comparison with Robinson [Ref. 2:p. 40], the values for the two ranks take opposite directions. Where the 1stLt FOM values are all

Table 6. COMBAT SUPPORT FIGURES OF MERIT

	1981	1982	1983
		TRANSFORMED FOM	
1st Lt			
MLE	2.1877	2.9028	2.2447
TSCA	1.6329	2.4580	1.8696
JS	1.6925	2.4816	1.8987
MOD JS	2.3684	3.0520	2.5356
EB	2.3323	3.1991	2.6868
MOD EB	2.2293	2.0048	2.1531
Lt Col			
MLE	1.3103	1.6978	2.3108
TSCA	0.7502	1.4070	1.7540
JS	0.6421	1.2345	1.5429
MOD JS	0.7009	1.0929	1.3082
EB	0.7736	1.2589	1.5799
MOD EB	1.8691	2.1236	4.9017
		ORIGINAL FOM	
1st Lt			
MLE	111.4260	112.1917	70.2767
TSCA	73.5008	90.0523	73.9380
TSCAM	79.0947	93.8244	80.9764
JS	76.9945	89.8064	71.5890
JSM	429.8503	84.4175	65.2639
MOD JS	143.1700	113.0083	90.6783
MOD JSM	168.9385	114.7083	92.6599
EB	102.9525	119.0900	96.3373
EBM	108.3578	111.2701	86.8138
MOD EB	68.7251	63.9054	48.2719
MOD EBM	176.3961	59.5760	43.0037
Lt Col			
MLE	36.0152	66.0261	42.4097
TSCA	61.7769	52.0346	99.7917
TSCAM	65.9535	49.6307	107.1016
JS	61.0511	49.2280	96.7861
JSM	66.3509	47.8490	118.2564
MOD JS	66.5470	49.9366	99.2566
MOD JSM	81.4722	50.2637	103.8460
EB	69.7508	55.9812	104.5311
EBM	71.5728	50.0281	109.2012
MOD EB	62.6316	50.3125	96.8693
MOD EBM	83.1337	48.6576	127.1541

larger (except for MLE in 1981),the LtCol values are smaller. Overall, all estimates are competitive with each other over all validation years with TSCA performing best for the 1stLt's and James-Stein outperforming the rest in the LtCol's. Note here also that the Mod EB estimate did not fair well in either case, and was the worst estimate in risk for the LtCol's.

In the original scale , a different and more random effect is noticed between Table 6 and the results of Robinson. Risk values tended to fluctuate with forecast lead time between estimates for 1stLt's where the LtCol values were more stable, yet were quite different from the transformed scale pattern. In this case, Mod EB performed well for 1stLt's, and was the only one within the desired range of standard deviations. MLE for the LtCol's was also the only estimate desirable for this measure. The Miller inverse showed poorly in this aggregate for all estimation methods, in every case exceeding it's counterpart.

In the MAD measure of effectiveness, Table 7, MLE performed the best in both ranks. The Mod EB effect is competitive in the 1stLt's case, and all variables in the LtCol case are closely grouped. The number of cells are heavily weighted towards an overage forecast in all three validation years, throughout this aggregate. Once again, this is not a desirable quality for a MOE.

Table 7. MEAN ABSOLUTE DEVIATION (COMBAT SUPPORT)

	1981	1982	1983
1st Lt			
MLE			
UNDERAGE	0.3737(14)	0.1617(8)	0.1781(8)
OVERAGE	0.6731(19)	0.9574(25)	0.8835(25)
TOTAL	1.0468(33)	1.1191(33)	1.0616(33)
MOD EB			
UNDERAGE	0.4028(15)	0.1275(9)	0.1427(9)
OVERAGE	0.7311(40)	0.9972(45)	0.9422(38)
TOTAL	1.1339(55)	1.1246(54)	1.0849(47)
MOD EBM			
UNDERAGE	0.4592(16)	0.1603(9)	0.1856(10)
OVERAGE	0.5930(39)	0.8318(45)	0.7575(37)
TOTAL	1.0522(55)	0.9921(54)	0.9431(47)
Lt Col			
MLE			
UNDERAGE	0.1360(9)	0.1688(7)	0.0541(5)
OVERAGE	0.1510(14)	0.2591(16)	0.3415(18)
TOTAL	0.2870(23)	0.4280(23)	0.3957(23)
MOD EB			
UNDERAGE	0.1638(11)	0.1721(7)	0.1162(6)
OVERAGE	0.2232(29)	0.3606(35)	0.4548(29)
TOTAL	0.3870(40)	0.5327(42)	0.5710(35)
MOD EBM			
UNDERAGE	0.2044(11)	0.1901(7)	0.1408(6)
OVERAGE	0.1263(29)	0.2240(35)	0.2589(29)
TOTAL	0.3307(40)	0.4142(42)	0.3996(35)

3. Ground Combat Aggregate Case

In the final aggregate studied by Robinson [Ref. 2:p. 41], that of ground combat MOS's, the results again, as displayed in Table 8, were mixed between computational methods. In the original scale, value differences tended to be larger than Robinson's in all cases for 1stLt's, showing a proportional growth over time. Conversely, LtCol values were smaller in the first two validation years and slightly larger in the final year. TSCA looks good for the 1stLt's with the others being competitive, yet the size of the values may not be desirable. Mod EB turned out the best for LtCol's, but all other estimates are competitive and in the desired range.

On the original scale, all values for the estimates for 1stLt's are too large, with Mod EB performing well in that regard. LtCol values are all larger than Robinson's, but the estimates are all within range of one standard deviation and, therefore, competitive. MLE, once again, shows well for this rank. In opposing fashion to that of the combat support aggregate, the FTE inversion outperformed all Basic inversions for all variables.

The MAD measure of effectiveness, Table 9 confirms the results found above in that Mod EB was best for 1stLt's and MLE did well for LtCol's. The values for 1stLt were found to be too large overall, while all estimates for LtCol were competitive and in the desired range. The mix between

Table 8. GROUND COMBAT FIGURES OF MERIT

	1981	1982	1983
		TRANSFORMED FOM	
1st Lt			
MLE	2.5764	4.4136	6.8744
TSCA	1.9741	3.8072	6.1152
JS	2.1737	3.9738	6.3970
MOD JS	3.0859	4.7919	7.5619
EB	2.5558	4.3626	6.9607
MOD EB	3.1538	3.5202	4.7069
Lt Col			
MLE	1.2126	1.9674	3.1084
TSCA	1.5715	2.3553	3.5566
JS	1.5571	2.2719	3.4365
MOD JS	1.6227	2.1317	3.1883
EB	1.7087	2.2464	3.3345
MOD EB	1.1964	2.5830	2.5029
		ORIGINAL FOM	
1st Lt			
MLE	87.3362	119.2637	143.9128
TSCA	87.9712	116.6767	145.6236
TSCAM	83.7648	111.3995	142.0902
JS	95.4717	122.9425	152.7354
JSM	92.0238	117.5264	149.3965
MOD JS	124.6490	147.6611	180.5502
MOD JSM	134.9472	142.5461	178.6173
EB	109.6349	136.6321	167.3234
EBM	105.5708	131.0400	163.8189
MOD EB	64.3647	81.2071	101.9511
MOD EBM	63.6623	77.8159	100.7150
Lt Col			
MLE	35.0538	45.8172	53.0111
TSCA	55.9649	198.8659	123.1306
TSCAM	43.5234	201.9953	122.8648
JS	56.3346	198.9037	122.2786
JSM	43.9957	201.3299	115.7143
MOD JS	60.0730	202.2959	122.9959
MOD JSM	52.3940	210.7551	121.4254
EB	62.7441	205.4249	126.6544
EBM	50.4049	209.3853	125.7642
MOD EB	48.0871	197.7316	111.9375
MOD EBM	38.1231	323.5654	242.4398

over-estimates and under-estimates, i.e., the number of cells forecasted over or under the actuals, is not a desirable one, yet the same mix holds true for all validation years. This also was the finding of both preceding aggregates.

Table 9. MEAN ABSOLUTE DEVIATION (GROUND COMBAT)

	1981	1982	1983
1st Lt			
MLE			
UNDERAGE	0.1525(5)	0.1855(10)	0.2466(10)
OVERAGE	1.9322(30)	2.5979(25)	3.1691(25)
TOTAL	2.0847(35)	2.7834(35)	3.4157(35)
MOD EB			
UNDERAGE	0.2340(10)	0.1323(8)	0.2220(8)
OVERAGE	1.7743(33)	2.3086(32)	2.8449(30)
TOTAL	2.0083(43)	2.4409(40)	3.0669(38)
MOD EBM			
UNDERAGE	0.2618(10)	0.1705(10)	0.2617(10)
OVERAGE	1.6492(33)	2.1738(30)	2.6935(28)
TOTAL	1.9111(43)	2.3443(40)	2.9552(38)
Lt Col			
MLE			
UNDERAGE	0.2313(10)	0.3034(11)	0.1368(10)
OVERAGE	0.4182(25)	0.4202(24)	0.6453(25)
TOTAL	0.6495(35)	0.7236(35)	0.7821(35)
MOD EB			
UNDERAGE	0.2315(11)	0.3299(14)	0.1426(12)
OVERAGE	0.6392(34)	0.6922(32)	0.9523(31)
TOTAL	0.8706(45)	1.0221(46)	1.0950(43)
MOD EBM			
UNDERAGE	0.2650(12)	0.3790(15)	0.1864(12)
OVERAGE	0.4071(33)	0.4427(31)	0.6535(31)
TOTAL	0.6720(45)	0.8217(46)	0.8399(43)

4. Test Aggregate Case

In an effort to experiment with aggregate groupings, another case was built upon which all estimation methods were applied, and the results compared with previously studied groups. The grade was held fixed, in this case the grade of Captain was used, with a LOS in the eighth year. Several MOS's were chosen so that to insure an aggregate with non-zero inventories. In the transformed scale, Table 10, all estimates have small and competitive values, with Mod EB and MLE doing well. In the original scale, once again, all estimates are well within range, but Mod EB and MLE were the standouts. The FTE inversion option did not fair well with this aggregate. All estimates in the MAD measure of effectiveness, Table 11, are very competitive, and the first validation year mix (overage and underage) may be the best yet. It is interesting to note that the cell split in the first validation year between overage and underage forecasts is contrary to that of the historical displays above. Although the total number of cells used (k=6) is much smaller than the previous table, it is felt that the aggregate scheme was the major factor in this finding, and the results are promising.

Table 10. TEST AGGREGATE FIGURES OF MERIT

	1981	1982	1983
	TRANSFORMED FOM		
Captain			
MLE	0.9603	2.3463	2.0842
TSCA	1.4151	3.3033	2.9152
JS	1.3753	3.3969	2.8725
MOD JS	1.3943	3.8165	2.8829
EB	1.6840	4.2698	3.2957
MOD EB	0.8249	1.7132	1.2980
	ORIGINAL FOM		
Captain			
MLE	3.0402	4.8104	4.9399
TSCA	4.6161	10.2779	7.5714
TSCAM	8.1467	7.6241	8.8347
JS	4.3970	10.3000	7.4726
JSM	7.3199	7.6666	8.3860
MOD JS	4.2990	10.7640	7.6371
MOD JSM	6.0079	8.3070	7.9259
EB	5.3660	12.0337	8.8679
EBM	6.7181	9.3915	8.7959
MOD EB	1.4463	6.0323	3.9066
MOD EBM	4.6465	4.3354	5.9175

Table 11. MEAN ABSOLUTE DEVIATION (TEST CASE)

	1981	1982	1983
Captain			
MLE			
UNDERAGE	0.2625(3)	0.0463(1)	0.2097(2)
OVERAGE	0.1351(3)	0.7374(5)	0.4391(4)
TOTAL	0.3976(6)	0.7837(6)	0.6488(6)
MOD EB			
UNDERAGE	0.1342(4)	0.0000(0)	0.0965(1)
OVERAGE	0.2575(2)	0.9601(6)	0.6306(5)
TOTAL	0.3918(6)	0.9601(6)	0.7270(6)
MOD EBM			
UNDERAGE	0.3712(4)	0.0809(1)	0.2352(2)
OVERAGE	0.0852(2)	0.6148(5)	0.3025(4)
TOTAL	0.4563(6)	0.6957(6)	0.5377(6)

V. CONCLUSIONS AND RECOMMENDATIONS

A. RESULTS

Omitting the fact of differences in computational methods between this paper and the Robinson thesis, it is clear that the results are still inconclusive at this time. In the historical comparisons, the estimates studied varied in their performance from one aggregate to another. The different risk calculations, in the transformed and original scales, as well as the different measures of effectiveness showed little agreement in estimation methods. Long range estimation still remains a problem since with time, the estimate risks increase significantly. Much emphasis was put on the modified empirical Bayes estimator in this paper and those specific results look promising under possibly different and more stable aggregate conditions.

One area of mention here concerning the results has to do with the composition of inventories used in the selected aggregates. It was noticed that the range of inventory (smallest to largest value in any particular MOS) was quite large, and may be affecting the results. In fact, Carter and Rolph [Ref. 4:p. 883] managed this point by further disaggregating into levels of activity as it concerned their work with fire alarm estimation.

B. CONCLUSIONS

The results of the previous feasibility studies and this paper leads to several conclusions. The chosen aggregate still seems to be the key element in the work of attrition estimation. It does seem clear that the aggregation scheme should be focused on fixed ranks, in narrow LOS ranges and multiple MOS's. This conclusion is based upon the relative success of the test aggregate utilized in this work, and upon similar success by Carter and Rolph. Additionally, the partitioning of inventory levels in the aggregate should stabilize the estimation method performance. All the estimation method studied to date have their strengths, but still, no one method of estimation excels over the others.

C. RECOMMENDATIONS

No estimation method studied in this paper is recommended for implementation at this time. In addition, no method is recommended for removal from consideration since basic aggregation problems are felt to be the largest contributory factor to the mixed results found here. These recommendations are presented for further study.

1. The aggregation method offered by Captain Larsen [Ref. 3] should be examined further with its relevance to the scheme in the test case used here.

2. Utilization of the detailed data provided by NPRDC as soon as possible in order to expand the available information and open new avenues of aggregation.

3. Re-examine the inventory mean value used in the FTE inverse to determine its future role in the estimation process.

APPENDIX A. ESTIMATION ALGORITHMS

A. NOTATION

The following notation is a generalization of that used by Robinson [Ref. 2:p. 71] as it pertains to the aggregation scheme used in this paper. The indexing system has been changed to reflect the use of officer grades to identify the cells.

i = index of MOS cells in the aggregate.

j = index of LOS cells in the aggregate.

k = index of Grade cells in the aggregate.

t = index of time periods in the aggregate.

inv(i,j,k,t) = inventory with MOS i, LOS j and Grade k during year t.

y(i,j,k,t) = number of attritions in cell (i,j,k) during year t.

n(i,j,k,t) = maximum {y(i,j,k,t),.5[inv(i,j,k,t) + inv(i,j,k,t + 1)]}

D = matrix identifying cells with no inventory over all estimation years.

d(i,j,k) = 0, if cell is a structural zero.

d(i,j,k) = 1, if cell is not a structural zero.

B. FREEMAN-TUKEY EXACT INVERSION ALGORITHM

This algorithm provides a formula for the inverse of the Freeman-Tukey double arcsine transformation to express the means of the transformed values into proportions on the original scale.

STEP 1. Transform the data using the Freeman-Tukey double arcsine transformation.

$$x(i,j,k,t) = 0.5[n(i,j,k,t)+0.5]^{1/2}\{\sin^{1}[\frac{2(y(i,j,k,t))}{(n(i,j,k,t)+1)} -1] + \sin^{-1}[\frac{2(y(i,j,k,t)+1}{(n(i,j,k,t)+1)} -1]\} \quad (A.1)$$

STEP 2. Calculate the average cell inventory and transformed values over all T.

$$\bar{x}(i,j,k) = (1/T)\sum_{t} x(i,j,k), \quad (A.2)$$

$$\overline{inv}(i,j,k) = (1/T)\sum_{t} inv(i,j,k). \quad (A.3)$$

STEP 3. Check for inversion values less than zero or greater than one. The algorithm is terminated at this point if these conditions are met.

$$tm = \bar{x}(i,j,k)/(\overline{inv}(i,j,k)+0.5)^{1/2} + \pi/2, \quad (A.4)$$

$$tm_1 = \sin^{-1}(1/(\overline{inv}(i,j,k) + 1))^{1/2}, \quad (A.5)$$

If $tm < tm_1$, FTE = 0.0,

IF $tm > (\pi - tm_1)$, FTE = 1.0,

STEP 4. Compute the sgn function.

If cos(tm) > 0.0 sgn = 1.0,

If cos(tm) < 0.0 sgn = -1.0.

STEP 5. Computer the inverted value of the transformed variable

$$FTE = 0.5(1-\mathrm{sgn}\{1-[\sin(tm) + \frac{\sin(tm) - 1/\sin(tm)}{\overline{inv}(i,j,k)}]^2\}^{1/2}) \qquad (A.6)$$

C. TRANSFORM SCALE VARIANCE ALGORITHM

This algorithm produces the variance of the transformed data within the unstable variance range where the assumption of normality does not hold. As discussed in Chapter II, this range of instability was determined to be between 1.001 and 2.2. The method for computing the variance within this range is presented here.

STEP 1. Compute the transformed scale values to be used for the variance. Since x(i,j,k) was computed in the FTE inversion algorithm of this appendix, the formulae will not be repeated.

$$\overline{x}(t) = \sum_{ijk} [0.5 + inv(i,j,k,t)]^{1/2}, \qquad (A.7)$$

$$z(i,j,k) = \overline{x}(i,j,k) + \frac{x(t)}{T}(\pi/2). \qquad (A.8)$$

STEP 2. Compute the variance of transformed cell values. The formula is derived in Chapter II as a linear regression equation. The range of the variance is limited to $0.05 \le var(i,j,k) \le 1.0$.

$$var(i,j,k) = az(i,j,k)^{b1}(z(i,j,k) - 1)^{b2}, \quad 1.001 \le z(i,j,k) \le 2.2 \tag{A.9}$$

where a = 1.6835, b1 = -0.8934, and b2 = 0.8991.

D. EMPIRICAL BAYES SHRINKAGE ALGORITHM

This algorithm computes the empirical Bayes shrinkage values and estimator of attrition rates. Prior to entering this algorithm, the value for a has been set to 0.0.

STEP 1. Initialization.

$$ai = a. \tag{A.10}$$

STEP 2. Define the α and γ parameters in the empirical Bayes context.

$$\alpha = \sum_{ijk} 1/(a + var(i,j,k)), \tag{A.11}$$

where var(i,j,k) is defined by formula (A.9).

$$\gamma = \sum_{ijk} \alpha / \sum_{ijk} 1/(\alpha + var(i,j,k)). \tag{A.12}$$

STEP 3. Compute the empirical Bayes prior variance.

$$a = a - \frac{k-1-\sum_{ijk} \alpha[\bar{x}(i,j,k)-\bar{z}]^2}{\sum_{ijk} [\alpha(\bar{x}(i,j,k) - \bar{z}]^2}, \tag{A.13}$$

where $z = \sum_{ijk} \gamma\bar{x}(i,j,k)$ and $k = \sum_{ijk} d(i,j,k)$.

STEP 4. Check for the following conditions and branch accordingly.

If, $a < 0.0$ set $a = 0.0$ and go to step 5,

If, $|a - ai| > .0001$ return to Step 1.

STEP 5. Compute the empirical Bayes estimator.

$$x^{EB}(i,j,k) = [\frac{a}{a+var(i,j,k)}]\bar{x}(i,j,k) + [\frac{var(i,j,k)}{a+var(i,j,k)}]\bar{z}. \tag{A.14}$$

STEP 6. Compute the empirical Bayes shrinkage set.

$$shr^{EB}(i,j,k) = var(i,j,k)/(a + var(i,j,k)). \tag{A.15}$$

E. MODIFIED EMPIRICAL BAYES SHRINKAGE ALGORITHM

This algorithm differs only slightly from that of the empirical Bayes shrinkage algorithm listed above, and for that reason, only the pertinent steps will be displayed here.

Consider a particular cell and its transformed value x(t) for t=1,...,T in the estimation set. To form a weighted time average, set

$$xt(t) = x(t)/[0.5 + inv(t)]^{1/2} \tag{A.16}$$

This leads to a modification of step 1 in the following form.

STEP 1. Compute the weighted transformed scale average.

$$\overline{xt} = \frac{1}{T} \sum_t xt(t) \tag{A.17}$$

Since the weights will modify the variance of the weighted transformed scale values.

$$var[xt(t)] = var(i,j,k)/(0.5 + inv(t)) \tag{A.18}$$

$$var[\overline{xt}(t)] = (\frac{1}{T})^2 \sum_t var[xt(t)] = vt/T \tag{A.19}$$

The extreme right of formula A.19 serves to define vt, the approximation for the weighted variance in the validation set for that cell.

The values of $\overline{xt}$ and var($\overline{xt}$) are carried throughout this version of empirical Bayes estimation to form the values XTEB for the empirical Bayes attrition rate. For the purposes of inversion to the original scale, the usual Basic formula is modified since the weighting factor, 0.5 + inv, is already incorporated into the transform value. The new Basic inversion for this version is simply as follows:

$$p = 1/2[1 + \sin(XTEB)] \tag{A.20}$$

APPENDIX B. MARINE CORPS PERSONNEL INVENTORY ATTRITION ANALYSIS PROGRAM

```
      PROGRAM MARCOR                                                    MAR00010
************************************************************************MAR00020
*     MARINE CORPS PERSONNEL INVENTORY ANALYSIS PROGRAM                *MAR00030
*                                                                      *MAR00040
*     PURPOSE: TO PROVIDE FUTURE PERSONNEL LOSS ESTIMATION OF THE      *MAR00050
*              MARINE CORPS OFFICER STRUCTURE BY SEVERAL DIFFERENT     *MAR00060
*              ESTIMATION SCHEMES.                                     *MAR00070
*     PROGRAMMERS: LUIS URIBE, INDEPENDENT CONTRACTOR                  *MAR00080
*                  CAPT C R DICKINSON, USMC                            *MAR00090
*     DESCRIPTION: WRITTEN IN FORTRAN 77 FOR THE IBM 3033 MAINFRAME    *MAR00100
*              COMPUTER RESIDENT AT NPS, THIS PROGRAM INCORPORATES     *MAR00110
*              METHODS EXPLORED BY MAJOR D.D. TUCKER , MAJOR J.R.      *MAR00120
*              ROBINSON AND CAPTAIN DICKINSON. ESTIMATION SCHEMES      *MAR00130
*              INCLUDED ARE:                                           *MAR00140
*                          MAXIMUM LIKELIHOOD ESTIMATION (MLE)         *MAR00150
*                          TRANSFORMED SCALE CELL AVERAGE (TSCA)       *MAR00160
*                          JAMES-STEIN                                 *MAR00170
*                          LIMITED TRANSLATION JAMES-STEIN             *MAR00180
*                          EMPIRICAL BAYES ESTIMATION                  *MAR00190
*              RISK ANALYSIS WAS PERFORMED IN BOTH THE ORIGINAL AND    *MAR00200
*              TRANSFORMED SCALES. ON THE TRANSFORMED SCALE,           *MAR00210
*              ACHIEVED BY USING THE FREEMAN-TUKEY DOUBLE ARCSINE      *MAR00220
*              TRANSFORMATION, THE RISK WAS DEFINED BY THE AVERAGE     *MAR00230
*              SQUARED DEVIATION OF ACTUAL AND ESTIMATED VALUES.       *MAR00240
*              FOR THE ORIGINAL SCALE, THE TRANSFORMED VALUES WERE     *MAR00250
*              FIRST INVERTED USING TWO TECHNIQUES. AN AD-HOC          *MAR00260
*              FORMULA USED BY TUCKER AND ROBINSON AND AN EXACT        *MAR00270
*              INVERSE FORMULA FROM JOHN J MILLER. TWO MEASURES OF     *MAR00280
*              EFFECTIVENESS, CHI-SQUARE GOODNESS OF FIT AND MEAN      *MAR00290
*              ABSOLUTE DEVIATION (MAD), WERE THEN EVALUATED.          *MAR00300
*              RESULTS ARE PRINTED, WITH ALL ARRAYS COMPUTED, AT THE   *MAR00310
*              COMPLETION OF THE PROGRAM.                              *MAR00320
*     VARIABLE DESCRIPTION:                                            *MAR00330
*              THE FOLLOWING LIST OF VARIABLES, WITH A BRIEF           *MAR00340
*              DESCRIPTION, FORM THE BASIS FOR THE VARIABLE NAMES      *MAR00350
*              FOR THE ENTIRE PROGRAM.                                 *MAR00360
*                    MLE - MAXIMUM LIKELIHOOD ESTIMATOR                *MAR00370
*                   TSCA - TRANSFORMED SCALE CELL AVERAGE              *MAR00380
*                      J - JAMES-STEIN ESTIMATOR                       *MAR00390
*                     J1 - JAMES-STEIN ESTIMATOR MODIFIED BY           *MAR00400
*                          A VARIATION ON SHRINKAGE                    *MAR00410
*                    JLT - LIMITED TRANSLATION JAMES-STEIN             *MAR00420
*                     EB - EMPIRICAL BAYES                             *MAR00430
*                    TEB - EMPIRICAL BAYES MODIFIED BY A               *MAR00440
*                          VARIATION ON                                *MAR00450
*        TRANSFORMED SPACE VARIABLES                                   *MAR00460
*              THE FOLLOWING ARE UNIQUE VARIABLES IN THE TRANSFORMED   *MAR00470
*              SPACE:                                                  *MAR00480
*                      X - VALUE OF THE FREEMAN-TUKEY                  *MAR00490
```

```
*                   TRANSFORMATION                              *  MAR00500
*              XB - (X-BAR) AVERAGE TRANSFORM VALUE             *  MAR00510
*                   OVER ESTIMATION YEARS                       *  MAR00520
*             XBB - GRAND MEAN OF TRANSFORMED VALUES            *  MAR00530
*             SSE - SUM OF SQUARED DIFFERENCES, ERROR           *  MAR00540
*                   (X AND X-BAR)                               *  MAR00550
*             SST - SUM OF SQUARED DIFFERENCES, TOTAL           *  MAR00560
*                   (X AND XBB)                                 *  MAR00570
*             SSB - SUM OF SQUARED DIFFERENCES, BETWEEN         *  MAR00580
*                          (SST-SSE)                            *  MAR00590
*                    SHR - JAMES-STEIN SHRINKAGE VALUES         *  MAR00600
*                   SHR1 - MODIFIED JAMES-STEIN SHRINKAGE       *  MAR00610
*                          VALUES                               *  MAR00620
*                  SHREB - EMPIRICAL BAYES SHRINKAGE VALUES     *  MAR00630
*                 SHRTEB - MODIFIED EMPIRICAL BAYES SHRINKAGE   *  MAR00640
*                          VALUES                               *  MAR00650
*       THE FOLLOWING ARE MODIFIED BASIC VARIABLES IN THE       *  MAR00660
*       TRANSFORMED SPACE. THIS IS DONE BY PLACING AN 'X' FOR   *  MAR00670
*       A PURE TRANSFORMED VARIABLE, OR AN 'R' FOR A RISK       *  MAR00680
*       VARIABLE IN FRONT OF THE BASIC VARIABLE.                *  MAR00690
*               MLE            RML (MLE)                        *  MAR00700
*               TSCA           RTSCA                            *  MAR00710
*               XJ             RJ                               *  MAR00720
*               XJ1            RJ1                              *  MAR00730
*               XJLT           RJLT                             *  MAR00740
*               XEB            RXEB                             *  MAR00750
*               XTEB           RTEB                             *  MAR00760
*    ORIGINAL SPACE VARIABLES                                   *  MAR00770
*       THE FOLLOWING ARE MODIFIED BASIC VARIABLES IN THE       *  MAR00780
*       ORIGINAL SPACE. SIMILAR TO WHAT IS DONE IN THE          *  MAR00790
*       TRANSFORMED SPACE, A 'P' SIGNIFIES A PURE INVERTED      *  MAR00800
*       VARIABLE, AND AN 'RR' REPRESENTS A RISK VARIABLE. AN    *  MAR00810
*       'M' AT THE END OF A VARIABLE NAME IDENTIFIES A MILLER   *  MAR00820
*       INVERSE VARIABLE.                                       *  MAR00830
*               PMLE           RRML (MLE)                       *  MAR00840
*               PJ             RRJ                              *  MAR00850
*               PJM            RRPJM                            *  MAR00860
*               PJ1            RRJ1                             *  MAR00870
*               PJ1M           RRJ1M                            *  MAR00880
*               PJLT           RRJLT                            *  MAR00890
*               PEB            RRXEB                            *  MAR00900
*               PEBM           RRXEBM                           *  MAR00910
*               PTEB           RRRTEB                           *  MAR00920
*               PTEBM          RRTEBM                           *  MAR00930
*       THE FOLLOWING LIST IS OF PREFIXES USED TO DESCRIBE      *  MAR00940
*       THE MEAN ABSOLUTE DEVIATION VARIABLES:                  *  MAR00950
*              MO - MEAN DEVIATION 'OVERAGE'                    *  MAR00960
*              MU - MEAN DEVIATION 'UNDERAGE'                   *  MAR00970
*              MD - MEAN 'TOTAL DEVIATION                       *  MAR00980
*             KVO - NUMBER OF 'OVERAGE' ESTIMATES IN THE        *  MAR00990
*                   AGGREGATE                                   *  MAR01000
*             KVU - NUMBER OF 'UNDERAGE' ESTIMATES IN THE       *  MAR01010
*                   AGGREGATE                                   *  MAR01020
*             KVD - NUMBER OF 'TOTAL' ESTIMATION DEVIATIONS     *  MAR01030
*                   IN THE AGGREGATE                            *  MAR01040
*       THESE PREFIXES ARE APPENDED TO THE BASIC VARIABLES TO   *  MAR01050
```

```
*              FORM THE MAD VARIABLES AS FOLLOWS                        *
*                  MOTSCA MUTSCA MDTSCA KVOTSA KVUTSA KVDTSA (TSCA)     *
*                  MOTSAM MUTSAM MDTSAM KVOTSM KVUTSM KVDTSM (TSCAM)    *
*                  MOPMLE MUPMLE MDPMLE KVOPML KVUPML KVDPML (PMLE)     *
*                  MOPJ   MUPJ   MDPJ   KVOPJ  KVUPJ  KVDPJ  (PJ)       *
*                  MOPJM  MUPJM  MDPJM  KVOPJM KVUPJM KVDPJM (PJM)      *
*                  MOPJ1  MUPJ1  MDPJ1  KVOPJ1 KVUPJ1 KVDPJ1 (PJ1)      *
*                  MOPJ1M MUPJ1M MDPJ1M KVOP1M KVUP1M KVDP1M (PJ1M)     *
*                  MOPEB  MUPEB  MDPEB  KVOPEB KVUPEB KVDPEB (PEB)      *
*                  MOPEBM MUPEBM MDPEBM KVOPBM KVUPBM KVDPBM (PEBM)     *
*                  MOPTEB MUPTEB MDPTEB KVOPTB KVUPTB KVDPTB (PTEB)     *
*                  MOPTBM MUPTBM MDPTBM KVOBTM KVUBTM KVDBTM (PTEBM)    *
*      INPUT/OUTPUT:                                                    *
*              INPUT IS TAKEN FROM A FILE CREATED FROM A TAPE           *
*                  PROVIDED BY NPRDC                                    *
*              OUTPUT IS TO THE PRINTER                                 *
*************************************************************************
**         FIXED PARAMETERS
      PARAMETER (MXYR=7, MXLOS=31, MXGRD=10, MXMOS=15)
**         INPUT PARAMETERS
      INTEGER ST1, ST2, LYR
      INTEGER SLOS1, SLOS2
      INTEGER SMOS(MXMOS), NMOS
      INTEGER SGRD(MXGRD), NGRD
**         VARIABLES IN ORIGINAL SPACE
      INTEGER T, FLAG, IVYR(MXYR)
      INTEGER*2  D(MXMOS, MXLOS, MXGRD)
      INTEGER KV(MXYR)
      REAL AVCINV,AINV,ATT,ATT1
      REAL CINV(MXMOS,MXLOS,MXGRD,MXYR),Y(MXMOS,MXLOS,MXGRD,MXYR)
      REAL ACINV(MXMOS,MXLOS,MXGRD)
      REAL PJ(MXMOS,MXLOS,MXGRD), PJM(MXMOS,MXLOS,MXGRD)
      REAL PMLE(MXMOS,MXLOS,MXGRD), PJLT(MXMOS,MXLOS,MXGRD)
      REAL PJ1(MXMOS,MXLOS,MXGRD), PJ1M(MXMOS,MXLOS,MXGRD)
      REAL PEB(MXMOS,MXLOS,MXGRD), PEBM(MXMOS,MXLOS,MXGRD)
      REAL PTEB(MXMOS,MXLOS,MXGRD), PTEBM(MXMOS,MXLOS,MXGRD)
      REAL RRTSCA(MXYR),RRJ(MXYR),RRJLT(MXYR),RRML(MXYR)
      REAL RRTSAM(MXYR),RRJM(MXYR),RRJ1(MXYR),RRJ1M(MXYR)
      REAL RRPEB(MXYR),RRPEBM(MXYR)
      REAL RRPTEB(MXYR),RRPTBM(MXYR)
**         VARIABLES IN TRANSFORMED SPACE
      REAL X(MXMOS,MXLOS,MXGRD,MXYR)
      REAL XT(MXMOS,MXLOS,MXGRD,MXYR)
      REAL XB(MXMOS,MXLOS,MXGRD), TSCA(MXMOS,MXLOS,MXGRD)
      REAL XTB(MXMOS,MXLOS,MXGRD)
      REAL XJ(MXMOS,MXLOS,MXGRD), XJLT(MXMOS,MXLOS,MXGRD)
      REAL XEB(MXMOS,MXLOS,MXGRD), XJ1(MXMOS,MXLOS,MXGRD)
      REAL XTEB(MXMOS,MXLOS,MXGRD)
      REAL XMLE(MXMOS,MXLOS,MXGRD)
      REAL SHREB(MXMOS,MXLOS,MXGRD)
      REAL SHRTEB(MXMOS,MXLOS,MXGRD)
      REAL RML(MXYR), RSL(MXYR),RXEB(MXYR)
      REAL RXTEB(MXYR)
      REAL RTSCA(MXYR),RTJ(MXYR),RTJLT(MXYR)
      REAL RJ(MXYR),RJLT(MXYR), RJ1(MXYR)
      REAL TSCAM(MXMOS,MXLOS,MXGRD)
```

```
      REAL V(MXMOS,MXLOS,MXGRD)
      REAL VT(MXMOS,MXLOS,MXGRD)
      REAL*8 SHR, SHRX, SHR1, SSB, SST, SSE, XBB
**       VARIABLES FOR FIGURE OF MERIT
      INTEGER KVOTSA(MXYR),KVOTSM(MXYR),KVOPJ(MXYR),KVOPJ1(MXYR)
      INTEGER KVOPJM(MXYR),KVOPEB(MXYR),KVOP1M(MXYR),KVOPBM(MXYR)
      INTEGER KVOPTB(MXYR),KVOBTM(MXYR)
      INTEGER KVOPML(MXYR)
      INTEGER KVUTSA(MXYR),KVUTSM(MXYR),KVUPJ(MXYR),KVUPJ1(MXYR)
      INTEGER KVUPJM(MXYR),KVUPEB(MXYR),KVUP1M(MXYR),KVUPBM(MXYR)
      INTEGER KVUPTB(MXYR),KVUBTM(MXYR)
      INTEGER KVUPML(MXYR)
      INTEGER KVDTSA(MXYR),KVDTSM(MXYR),KVDPJ(MXYR),KVDPJ1(MXYR)
      INTEGER KVDPJM(MXYR),KVDPEB(MXYR),KVDP1M(MXYR),KVDPBM(MXYR)
      INTEGER KVDPTB(MXYR),KVDBTM(MXYR)
      INTEGER KVDPML(MXYR)
      REAL MUTSCA(MXYR),MUTSAM(MXYR),MUPJ(MXYR),MUPJ1(MXYR),MUPJM(MXYR)
      REAL MUPJ1M(MXYR),MUPEBM(MXYR),MUPEB(MXYR),MUPMLE(MXYR)
      REAL MUPTBM(MXYR),MUPTEB(MXYR)
      REAL MOTSCA(MXYR),MOTSAM(MXYR),MOPJ(MXYR),MOPJ1(MXYR),MOPJM(MXYR)
      REAL MOPJ1M(MXYR),MOPEBM(MXYR),MOPEB(MXYR),MOPMLE(MXYR)
      REAL MOPTBM(MXYR),MOPTEB(MXYR)
      REAL MDTSCA(MXYR),MDTSAM(MXYR),MDPJ(MXYR),MDPJ1(MXYR),MDPJM(MXYR)
      REAL MDPJ1M(MXYR),MDPEBM(MXYR),MDPEB(MXYR),MDPMLE(MXYR)
      REAL MDPTBM(MXYR),MDPTEB(MXYR)
      REAL MILLER
**        INPUT DATA AREAS
      INTEGER MOS, LOS, GRADE, YR, INV
      INTEGER MOS1,LOS1,GRADE1,YR1,INV1
**        INPUT INITIALIZATION
      DATA KV/MXYR*0/
      DATA MOPMLE/MXYR*0/,MUPMLE/MXYR*0/,MDPMLE/MXYR*0/
      DATA KVOPML/MXYR*0/,KVUPML/MXYR*0/,KVDPML/MXYR*0/
      DATA MOTSCA/MXYR*0/,MUTSCA/MXYR*0/,MDTSCA/MXYR*0/
      DATA KVOTSA/MXYR*0/,KVUTSA/MXYR*0/,KVDTSA/MXYR*0/
      DATA MOTSAM/MXYR*0/,MUTSAM/MXYR*0/,MDTSAM/MXYR*0/
      DATA KVOTSM/MXYR*0/,KVUTSM/MXYR*0/,KVDTSM/MXYR*0/
      DATA MOPJ/MXYR*0/,MUPJ/MXYR*0/,MDPJ/MXYR*0/
      DATA KVOPJ/MXYR*0/,KVUPJ/MXYR*0/,KVDPJ/MXYR*0/
      DATA MOPJM/MXYR*0/,MUPJM/MXYR*0/,MDPJM/MXYR*0/
      DATA KVOPJM/MXYR*0/,KVUPJM/MXYR*0/,KVDPJM/MXYR*0/
      DATA MOPJ1/MXYR*0/,MUPJ1/MXYR*0/,MDPJ1/MXYR*0/
      DATA KVOPJ1/MXYR*0/,KVUPJ1/MXYR*0/,KVDPJ1/MXYR*0/
      DATA MOPJ1M/MXYR*0/,MUPJ1M/MXYR*0/,MDPJ1M/MXYR*0/
      DATA KVOP1M/MXYR*0/,KVUP1M/MXYR*0/,KVDP1M/MXYR*0/
      DATA MOPEB/MXYR*0/,MUPEB/MXYR*0/,MDPEB/MXYR*0/
      DATA KVOPEB/MXYR*0/,KVUPEB/MXYR*0/,KVDPEB/MXYR*0/
      DATA MOPEBM/MXYR*0/,MUPEBM/MXYR*0/,MDPEBM/MXYR*0/
      DATA KVOPBM/MXYR*0/,KVUPBM/MXYR*0/,KVDPBM/MXYR*0/
      DATA MOPTEB/MXYR*0/,MUPTEB/MXYR*0/,MDPTEB/MXYR*0/
      DATA KVOPTB/MXYR*0/,KVUPTB/MXYR*0/,KVDPTB/MXYR*0/
      DATA MOPTBM/MXYR*0/,MUPTBM/MXYR*0/,MDPTBM/MXYR*0/
      DATA KVOBTM/MXYR*0/,KVUBTM/MXYR*0/,KVDBTM/MXYR*0/
*
**********************************************************************
```

```
**       READ PARAMETER DATA TO USE IN THIS RUN.                       **
************************************************************************
      WRITE(5,100)
100   FORMAT(' ENTER 1ST YEAR AND LAST YEAR TO USE FOR ESTIMATION')
      READ(5,*) ST1,ST2
      WRITE(6,*) 'ESTIMATION YEARS: ',ST1,ST2
*
      WRITE(5,101)
101   FORMAT(' ENTER LAST YEAR PRESENT IN THE DATA BASE')
      READ(5,*) LYR
      WRITE(6,*) 'LAST YEAR AVAILABLE FOR VALIDATION:',LYR
*
      WRITE(5,102)
102   FORMAT(' ENTER NO. OF MOS FOLLOWED BY ARRAY OF SAME LENGTH')
      READ(5,*) NMOS, (SMOS(I), I=1,NMOS)
      WRITE(6,*) 'MOS SELECTED:', (SMOS(I), I=1,NMOS)
*
      WRITE(5,103)
103   FORMAT(' ENTER 1ST AND LAST LOS VALUE TO USE')
      READ(5,*) SLOS1,SLOS2
      WRITE(6,*) 'LOS RANGE:',SLOS1,SLOS2
*
      WRITE(5,104)
104   FORMAT(' ENTER NO. OF GRADE FOLLOWED BY ARRAY OF SAME LENGTH')
      READ(5,*) NGRD, (SGRD(I), I=1,NGRD)
      WRITE(6,*) 'GRADES SELECTED', (SGRD(I), I=1,NGRD)
*
      WRITE(5,105)
105   FORMAT(' ENTER DEE FACTOR')
      READ(5,*) DEE
      WRITE(6,*) 'DEE FACTOR:',DEE
*
**       COMPUTE START AND END YEARS FOR VALIDATION
      NYR=LYR-ST1+1
      NVYR=LYR-ST2
      NLOS=SLOS2-SLOS1+1
      T=ST2-ST1+1
      DO 41 I=1,NVYR
       IVYR(I)=ST2+I
41    CONTINUE
************************************************************************
**       INITIALIZE D ARRAY TO ALLOW FINDING MISSING VALUES.           **
************************************************************************
      DO 1 I=1,MXMOS
       DO 2 J=1,MXLOS
        DO 3 K=1,MXGRD
         D(I,J,K)=-9999
3       CONTINUE
2      CONTINUE
1     CONTINUE
************************************************************************
**       TAPE PROCESSING TO INPUT INVENTORY AND ATTRITION VALUES,      **
**       DEVELOPE D MATRIX ACCORDING TO EXTRACTION CRITERIA.           **
************************************************************************
      FLAG=0
      IEOF=0
```

```
      CALL READER(MOS1,LOS1,GRADE1,YR1,INV1,ATT1, NMOS,SMOS,
     *       NGRD,SGRD, SLOS1,SLOS2, ST1,ST2, IM1,IL1,IG1,IT1,IEOF)
*
      IF(IEOF .NE. 0) THEN
              WRITE(6,*) '**** NO DATA MEETS SELECTIONS REQS'
              STOP
      ENDIF
*
11    IF(IEOF .NE. 0) GO TO 9
       MOS=MOS1
       LOS=LOS1
       GRADE=GRADE1
       YR=YR1
       INV=INV1
       ATT=ATT1
       IM=IM1
       IL=IL1
       IG=IG1
       IT=IT1
       IF(YR.GT.ST1) FLAG = 1
**       CHECK CASE WHERE DATA BEGINS PAST 1ST YR
       IF(YR .GT. ST1) CINV(IM,IL,IG,IT-1)=.5*FLOAT(INV)
       CALL READER(MOS1,LOS1,GRADE1,YR1,INV1,ATT1, NMOS,SMOS,
     *       NGRD,SGRD, SLOS1,SLOS2, ST1,ST2, IM1,IL1,IG1,IT1,IEOF)
**       ONLY FOR CHECKING PURPOSES. MARK D TO INDICATE SOME YR PRESENT
       D(IM,IL,IG)=0
*
12       IF(.NOT.(IEOF.EQ.0 .AND. MOS1.EQ.MOS .AND. LOS1.EQ.LOS
     *                      .AND. GRADE1.EQ.GRADE)) GO TO 8
**       CENTRAL INV. FOR YR1 - 1
        IF(YR1 .GT. YR+1) THEN
              CINV(IM1,IL1,IG1,IT1-1)=.5*FLOAT(INV1)
              AINV=.5*FLOAT(INV)
              I=YR+1
              FLAG=1
        ELSE
              AINV=.5*FLOAT(INV+INV1)
        END IF
        CINV(IM,IL,IG,IT)=AMAX1(AINV,ATT)
        Y(IM,IL,IG,IT)=ATT
        YR=YR1
        INV=INV1
        ATT=ATT1
        IM=IM1
        IL=IL1
        IG=IG1
        IT=IT1
        CALL READER(MOS1,LOS1,GRADE1,YR1,INV1,ATT1, NMOS,SMOS,
     *       NGRD,SGRD, SLOS1,SLOS2, ST1,ST2, IM1,IL1,IG1,IT1,IEOF)
        GO TO 12
8      CONTINUE
**       WHEN YEARS MISSING AT THE END
       IF(YR .LT. LYR) THEN
              FLAG=1
              AINV=.5*FLOAT(INV)
       ELSE
```

```
          AINV=FLOAT(INV)
     END IF
     CINV(IM,IL,IG,IT)=AMAX1(AINV,ATT)
     Y(IM,IL,IG,IT)=ATT

    CC=CINV(IM,IL,IG,IT)
    TEMP=-1.+2.*ATT/(1.+CC)
    TEMP1=-1. + 2.*(1.+ATT)/(1.+CC)
    CC=ABS(TEMP)
    CC1=ABS(TEMP1)
    IF(CC .GT. 1. .OR. CC1 .GT. 1.) WRITE(6,*) '*** TEMP, TEMP1=',
   *  TEMP,TEMP1,'***MOS,LOS,GR,YR=',SMOS(IM),IL,SGRD(IG),IT

     GO TO 11
    CONTINUE
*********************************************************************
*      CALCULATE AVERAGE CENTRAL INVENTORY AND MAXIMUM             **
*                  LIKELIHOOD ESTIMATE.                            **
*********************************************************************
    DO 10 IM=1,NMOS
     DO 20 IL=1,NLOS
      DO 30 IG=1,NGRD
       SUMY=0
       ACINV(IM,IL,IG)=0
       DO 35 IT=1,T
        SUMY=SUMY+Y(IM,IL,IG,IT)
        ACINV(IM,IL,IG)=ACINV(IM,IL,IG)+CINV(IM,IL,IG,IT)
5      CONTINUE
       IF(ACINV(IM,IL,IG) .NE. 0) THEN
                         PMLE(IM,IL,IG)=SUMY/ACINV(IM,IL,IG)
       ELSE
                         PMLE(IM,IL,IG)=0
       ENDIF
       ACINV(IM,IL,IG)=ACINV(IM,IL,IG)/T
       DO 40 IT=1,NYR
*      IF ANY CENTRAL INVENTORY>0 (OVER EST. YRS T) THEN D=1
        IF(CINV(IM,IL,IG,IT).GT.0 .AND. IT.LE.T) D(IM,IL,IG)=1
*      IF ANY CENTRAL INVENTORY>0 (OVER VAL. YRS > T) THEN COUNT
        IF(CINV(IM,IL,IG,IT).GT.0 .AND. IT.GT.T) KV(IT-T)=KV(IT-T)+1
*      TRANSFORMATION OF CINV USING THE FREEMAN-TUKEY
*               DOUBLE ARCSIN FORMULA.
        CALL FTT(CINV(IM,IL,IG,IT),Y(IM,IL,IG,IT),X(IM,IL,IG,IT))
        XT(IM,IL,IG,IT)=X(IM,IL,IG,IT) / SQRT(0.5+CINV(IM,IL,IG,IT))
0      CONTINUE
0     CONTINUE
0    CONTINUE
0   CONTINUE
*********************************************************************
*      CHECK FOR MISSING DATA, COMPUTE THE TRANSFORMED             **
*             CELL AVERAGES AND GRAND MEAN.                        **
*********************************************************************
    XBB=0
    KK=0
    DO 50 IM=1,NMOS
     DO 60 IL=1,NLOS
      DO 70 IG=1,NGRD
```

```
         IF(D(IM,IL,IG) .EQ. -9999) THEN
**       REPORT MISSING COMBINATION IM,IL,IG & CLEAR D ENTRY
                D(IM,IL,IG)=0
                ILD=SLOS1+IL-1
                FLAG=1
         END IF
         KK=KK+D(IM,IL,IG)
         XB(IM,IL,IG)=0
         XTB(IM,IL,IG)=0
         DO 80 IT=1,T
          XB(IM,IL,IG)=XB(IM,IL,IG)+X(IM,IL,IG,IT)
          XTB(IM,IL,IG)=XTB(IM,IL,IG)+XT(IM,IL,IG,IT)
80       CONTINUE
         XB(IM,IL,IG)=XB(IM,IL,IG)/T
         XTB(IM,IL,IG)=XTB(IM,IL,IG)/T
**    EMPIRICAL BAYES PREPARATION ON THE TRANSFORMED SCALE.
         XEB(IM,IL,IG)=XB(IM,IL,IG)
         XTEB(IM,IL,IG)=XTB(IM,IL,IG)
*
         XBB=XBB+D(IM,IL,IG)*XB(IM,IL,IG)
70      CONTINUE
60     CONTINUE
50    CONTINUE
      XBB=XBB/KK
************************************************************************
**       BAYES VARIANCE ITERATION SECTION.                             **
************************************************************************
      IC=0
**       COMPUTE TRANSFORMED SCALE CELL VARIANCE.
499   DO 500 IM=1,NMOS
        DO 510 IL=1,NLOS
          DO 520 IG=1,NGRD
            IF (D(IM,IL,IG).NE.1) GOTO 520
            AVSQR = 0.0
            DO 522 IT=1,T
522            AVSQR = AVSQR + SQRT(0.5 + CINV(IM,IL,IG,IT))
            Z = XEB(IM,IL,IG) + (AVSQR/T) * 1.5708
            V(IM,IL,IG) = VAR(Z)
520       CONTINUE
510     CONTINUE
500   CONTINUE
*
      A=0
530   AI=A
      SALPH=0
      DO 531 IM=1,NMOS
        DO 532 IL=1,NLOS
          DO 533 IG=1,NGRD
533         IF (D(IM,IL,IG).EQ.1) SALPH = SALPH + 1./(A + V(IM,IL,IG))
532     CONTINUE
531   CONTINUE
*
      ZB=0
**       COMPUTE BAYES ALPHA AND GAMMA PARAMETERS
      DO 537 IM=1,NMOS
        DO 538 IL=1,NLOS
```

```
        DO 539 IG=1,NGRD
          IF (D(IM,IL,IG).EQ.1) THEN
            ALPH = 1./(A + V(IM,IL,IG))
            GAM  = ALPH / SALPH
            ZB   = ZB + GAM*XB(IM,IL,IG)
          ENDIF
539     CONTINUE
538   CONTINUE
537 CONTINUE
          F=0
          G=0
    DO 540 IM=1,NMOS
      DO 541 IL=1,NLOS
        DO 542 IG=1,NGRD
          IF (D(IM,IL,IG).EQ.1) THEN
            ALPH = 1./(A + V(IM,IL,IG))
            F    = F + (ALPH * (XB(IM,IL,IG) - ZB)**2)
            G    = G + (ALPH * (XB(IM,IL,IG)-ZB))**2
          ENDIF
542     CONTINUE
541   CONTINUE
540 CONTINUE
**     COMPUTE THE PRIOR VARIANCE (A)
    A = A - (KK-1-F)/G
*
    IF (A.LE.0.) THEN
      A=0
      GOTO 550
    ENDIF
    IF (ABS(A-AI).GT..0001) GOTO 530
**     COMPUTE THE EMPIRICAL BAYES ATTRITION RATES
550 DO 570 IM=1,NMOS
      DO 580 IL=1,NLOS
        DO 590 IG=1,NGRD
          IF (D(IM,IL,IG).EQ.1) THEN
            DEN = A + V(IM,IL,IG)
            XEB(IM,IL,IG) = (A/DEN)*XB(IM,IL,IG)+((DEN-A)/DEN )*ZB
          ENDIF
590     CONTINUE
580   CONTINUE
570 CONTINUE
    IC=IC+1
    IF (A.EQ.0) GOTO 599
    IF (IC.LT.10) GOTO 499
*
599 CONTINUE
*
**     COMPUTE THE EMPIRICAL BAYES SHRINKAGE RATES
    DO 600 IM=1,NMOS
      DO 601 IL=1,NLOS
        DO 602 IG=1,NGRD
          SHREB(IM,IL,IG) = 0
          IF (D(IM,IL,IG).EQ.1) THEN
            SHREB(IM,IL,IG) = V(IM,IL,IG)/(A + V(IM,IL,IG))
          ENDIF
602     CONTINUE
```

```
601     CONTINUE
600   CONTINUE
*******************************************************************************
**        ALTERNATE BAYES VARIANCE ITERATION SECTION.                       **
*******************************************************************************
**        COMPUTE TRANSFORMED SCALE CELL VARIANCE.
699   DO 700 IM=1,NMOS
        DO 710 IL=1,NLOS
          DO 720 IG=1,NGRD
            VT(IM,IL,IG) = 0.0
            IF (D(IM,IL,IG).NE.1) GOTO 720
            DO 725 IT=1,T
              ZT = X(IM,IL,IG,IT) + 1.5708 * SQRT(0.5 +
     *             CINV(IM,IL,IG,IT))
725           VT(IM,IL,IG) = VT(IM,IL,IG) + VAR(ZT)/(0.5 +
     *                       CINV(IM,IL,IG,IT))
            VT(IM,IL,IG) = VT(IM,IL,IG)/T**2
720       CONTINUE
710     CONTINUE
700   CONTINUE
*
      AT=0
730   ATI=AT
      SALPHT=0
      DO 731 IM=1,NMOS
        DO 732 IL=1,NLOS
          DO 733 IG=1,NGRD
733         IF (D(IM,IL,IG).EQ.1) SALPHT= SALPHT+ 1./(AT+ VT(IM,IL,IG))
732     CONTINUE
731   CONTINUE
      ZTB=0
**        COMPUTE BAYES ALPHA AND GAMMA PARAMETERS
      DO 737 IM=1,NMOS
        DO 738 IL=1,NLOS
          DO 739 IG=1,NGRD
            IF (D(IM,IL,IG).EQ.1) THEN
              ALPHT= 1./(AT+ VT(IM,IL,IG))
              GAMT = ALPHT / SALPHT
              ZTB  = ZTB+ GAMT*XTB(IM,IL,IG)
            ENDIF
739       CONTINUE
738     CONTINUE
737   CONTINUE
      PZTB=0.5*(1.+SIN(ZTB))
      FT=0
      GT=0
      DO 740 IM=1,NMOS
        DO 741 IL=1,NLOS
          DO 742 IG=1,NGRD
            IF (D(IM,IL,IG).EQ.1) THEN
              ALPHT= 1./(AT + VT(IM,IL,IG))
              FT   = FT + (ALPHT * (XTB(IM,IL,IG) - ZTB)**2)
              GT   = GT + (ALPHT * (XTB(IM,IL,IG) - ZTB))**2
            ENDIF
742       CONTINUE
741     CONTINUE
```

```
740   CONTINUE
**       COMPUTE THE PRIOR VARIANCE (A)
      AT= AT- (KK-1-FT)/GT
*
      IF (AT.LE.0.) THEN
        AT=0
        GOTO 750
      ENDIF
      IF (ABS(AT-ATI).GT..0001) GOTO 730
**       COMPUTE THE EMPIRICAL BAYES ATTRITION RATES
750   DO 770 IM=1,NMOS
        DO 780 IL=1,NLOS
          DO 790 IG=1,NGRD
            IF (D(IM,IL,IG).EQ.1) THEN
              DENT = AT + VT(IM,IL,IG)
              XTEB(IM,IL,IG) = (AT/DENT)*XTB(IM,IL,IG) +
     *                         ((DENT-AT)/DENT)*ZTB
            ENDIF
790       CONTINUE
780     CONTINUE
770   CONTINUE
**       COMPUTE THE EMPIRICAL BAYES SHRINKAGE RATES
      DO 800 IM=1,NMOS
        DO 801 IL=1,NLOS
          DO 802 IG=1,NGRD
            SHRTEB(IM,IL,IG) = 0
            IF (D(IM,IL,IG).EQ.1) THEN
              SHRTEB(IM,IL,IG) = VT(IM,IL,IG)/(AT + VT(IM,IL,IG))
            ENDIF
802       CONTINUE
801     CONTINUE
800   CONTINUE
*************************************************************************
**         JAMES-STEIN SHRINKAGE RATES.                                **
*************************************************************************
      SST=0
      SSE=0
      SSB=0
**       COMPUTE THE SUMS OF SQUARES (SST, SSE AND SSB)
      DO 90 IM=1,NMOS
       DO 95 IL=1,NLOS
        DO 110 IG=1,NGRD
         TEMP1=0
         TEMP2=0
         DO 120 IT=1,T
          TEMP1=TEMP1+(X(IM,IL,IG,IT)-XBB)**2
          TEMP2=TEMP2+(X(IM,IL,IG,IT)-XB(IM,IL,IG))**2
120      CONTINUE
         SST=SST+D(IM,IL,IG)*TEMP1
         SSE=SSE+D(IM,IL,IG)*TEMP2
110     CONTINUE
95     CONTINUE
90    CONTINUE
      SSB=SST-SSE
**       COMPUTE THE JAMES-STEIN SHRINKAGE RATES
      SHRX=(SSE/SSB)* (KK-3)/(KK*(T-1)+2)
```

```
      SHR=DMIN1(SHRX, 1.D0)
      SHR=DMAX1(SHR , 0.D0)
      SHR1=T*SHR
*********************************************************************
**        LIMITED TRANSLATION OPTION & JAMES-STEIN                 **
**              ON TRANSFORMED SCALE.                              **
*********************************************************************
      NN=KK*(T-1)
      DO 125 IM=1,NMOS
       DO 130 IL=1,NLOS
        DO 140 IG=1,NGRD
         XJ(IM,IL,IG)=0
         XJ1(IM,IL,IG)=0
         IF (D(IM,IL,IG).EQ.1) THEN
           XJ(IM,IL,IG)=XBB+(1-SHR)*(XB(IM,IL,IG)-XBB)
           XJ1(IM,IL,IG)=XBB+(1-SHR1)*(XB(IM,IL,IG)-XBB)
C ***      XJLT(IM,IL,IG)=XBB+(1-SHR*RHO(XB(IM,IL,IG),XBB,SSE,
C ***                         SSB,DEE,KK,T,NN))*(XB(IM,IL,IG)-XBB)
           XMLE(IM,IL,IG)=SQRT(.5+ACINV(IM,IL,IG))
     *                                *ASIN(2*PMLE(IM,IL,IG)-1)
         ENDIF
140     CONTINUE
130    CONTINUE
125   CONTINUE
*********************************************************************
**        VALIDATION.  RISK EVALUATED IN TRANSFORM SPACE.          **
*********************************************************************
      DO 150 IT=1, NVYR
       RTSCA(IT)=0
       RJ(IT)=0
       RJ1(IT)=0
       RXEB(IT)=0
       RXTEB(IT)=0
C ***  RJLT(IT)=0
       RML(IT)=0
       DO 160 IM=1,NMOS
        DO 170 IL=1,NLOS
         DO 180 IG=1,NGRD
          IF(D(IM,IL,IG).EQ.1 .AND. CINV(IM,IL,IG,IT+T).NE.0) THEN
             RTSCA(IT)=RTSCA(IT)+(X(IM,IL,IG,T+IT)-  XB(IM,IL,IG))**2
                RJ(IT)=   RJ(IT)+(X(IM,IL,IG,T+IT)-  XJ(IM,IL,IG))**2
               RJ1(IT)= RJ1(IT)+(X(IM,IL,IG,T+IT)- XJ1(IM,IL,IG))**2
              RXEB(IT)=RXEB(IT)+(X(IM,IL,IG,T+IT)- XEB(IM,IL,IG))**2
              RXTEB(IT)=RXTEB(IT)+((X(IM,IL,IG,T+IT)/SQRT(0.5 +
     *                 CINV(IM,IL,IG,IT+T))- XTEB(IM,IL,IG))**2)
     *                 /VT(IM,IL,IG)
C ***        RJLT (IT)= RJLT(IT)+(X(IM,IL,IG,T+IT)-XJLT(IM,IL,IG))**2
             RML  (IT)= RML (IT)+(X(IM,IL,IG,T+IT)-XMLE(IM,IL,IG))**2
          ENDIF
180      CONTINUE
170     CONTINUE
160    CONTINUE
       RTSCA(IT)=RTSCA(IT)/KV(IT)
       RJ  (IT)=RJ  (IT)/KV(IT)
       RJ1 (IT)=RJ1 (IT)/KV(IT)
       RXEB(IT)=RXEB(IT)/KV(IT)
```

```
      RXTEB(IT)=RXTEB(IT)/(KV(IT)*T)
C *** RJLT(IT)=RJLT(IT)/KV(IT)
      RML (IT)=RML (IT)/KV(IT)
      RTJ(IT)=RTSCA(IT)/RJ(IT)
C *** RTJLT(IT)=RTSCA(IT)/RJLT(IT)
C *** RSL(IT)=(RJ(IT)-RTSCA(IT))/(RJLT(IT)-RTSCA(IT))
150  CONTINUE
*************************************************************************
**       DO RISK ANALYSIS ON THE ORIGINAL SCALE.                      **
*************************************************************************
     DO 200 IM=1,NMOS
      DO 210 IL=1,NLOS
       DO 220 IG=1,NGRD
**      AVERAGE CENTRAL INVENTORY OVER ESTIMATION SET
        CINVS=0
        DO 230 IT=1,T
        AVCINV=AVCINV + CINV(IM,IL,IG,IT)
230     CONTINUE
        AVCINV=AVCINV/T
        TSCA(IM,IL,IG)=D(IM,IL,IG)*SCINV(  XB(IM,IL,IG), AVCINV)
          PJ(IM,IL,IG)=D(IM,IL,IG)*SCINV(  XJ(IM,IL,IG), AVCINV)
         PJ1(IM,IL,IG)=D(IM,IL,IG)*SCINV( XJ1(IM,IL,IG), AVCINV)
         PEB(IM,IL,IG)=D(IM,IL,IG)*SCINV( XEB(IM,IL,IG), AVCINV)
         PTEB(IM,IL,IG)=D(IM,IL,IG)*0.5*(1. + SIN(XTEB(IM,IL,IG)))
C ***   PJLT(IM,IL,IG)=D(IM,IL,IG)*SCINV(XJLT(IM,IL,IG), AVCINV)
       IF (D(IM,IL,IG).EQ.1) THEN
         TSCAM(IM,IL,IG)=MILLER(  XB(IM,IL,IG), AVCINV )
           PJM(IM,IL,IG)=MILLER(  XJ(IM,IL,IG), AVCINV )
          PJ1M(IM,IL,IG)=MILLER( XJ1(IM,IL,IG), AVCINV )
          PEBM(IM,IL,IG)=MILLER( XEB(IM,IL,IG), AVCINV )
             XMIL = XTEB(IM,IL,IG) * SQRT(0.5 + AVCINV)
          PTEBM(IM,IL,IG)=MILLER( XMIL , AVCINV )
       ENDIF
220    CONTINUE
210   CONTINUE
200  CONTINUE
*************************************************************************
**      VALIDATION.  RISK EVALUATED IN ORIGINAL SPACE.                **
*************************************************************************
     DO 240 IT=1, NVYR
      RRTSCA(IT)=0
      RRTSAM(IT)=0
      RRJ(IT)=0
      RRJM(IT)=0
      RRPEB(IT)=0
      RRPTEB(IT)=0
      RRPEBM(IT)=0
      RRPTBM(IT)=0
      RRJ1(IT)=0
      RRJ1M(IT)=0
C *** RRJLT(IT)=0
      RRML(IT)=0
      KP=0
      DO 250 IM=1,NMOS
       DO 260 IL=1,NLOS
        DO 270 IG=1,NGRD
```

```
        CC=CINV(IM,IL,IG,T+IT)
        IF(CC .NE.0) THEN
                          AR=Y(IM,IL,IG,T+IT)/CC
        ELSE
                          AR=0
        ENDIF
        IF(D(IM,IL,IG) .NE. 0 .AND. PMLE(IM,IL,IG).NE.0
     *             .AND. PMLE(IM,IL,IG).NE.1) THEN
          KP=KP+1
          RRML(IT)=RRML(IT)+(CC*(AR-PMLE(IM,IL,IG))**2)
     *                      /(PMLE(IM,IL,IG)*(1. - PMLE(IM,IL,IG)))
        ENDIF
**     PRODUCE CHI-SQUARE MEASURE OF EFFECTIVENESS
        RRTSCA(IT)= RRTSCA(IT)+CHISQR(D(IM,IL,IG),TSCA(IM,IL,IG),
     *                                CC,AR)
        RRTSAM(IT)= RRTSAM(IT)+CHISQR(D(IM,IL,IG),TSCAM(IM,IL,IG),
     *                                CC,AR)
        RRJ   (IT)= RRJ   (IT)+CHISQR(D(IM,IL,IG),PJ  (IM,IL,IG),
     *                                CC,AR)
        RRJM  (IT)= RRJM  (IT)+CHISQR(D(IM,IL,IG),PJM (IM,IL,IG),
     *                                CC,AR)
        RRJ1  (IT)= RRJ1  (IT)+CHISQR(D(IM,IL,IG),PJ1 (IM,IL,IG),
     *                                CC,AR)
        RRJ1M (IT)= RRJ1M (IT)+CHISQR(D(IM,IL,IG),PJ1M(IM,IL,IG),
     *                                CC,AR)
        RRPEB (IT)= RRPEB (IT)+CHISQR(D(IM,IL,IG),PEB (IM,IL,IG),
     *                                CC,AR)
        RRPTEB (IT)= RRPTEB (IT)+CHISQR(D(IM,IL,IG),PTEB (IM,IL,IG),
     *                                CC,AR)
        RRPEBM(IT)= RRPEBM(IT)+CHISQR(D(IM,IL,IG),PEBM(IM,IL,IG),
     *                                CC,AR)
        RRPTBM(IT)= RRPTBM(IT)+CHISQR(D(IM,IL,IG),PTEBM(IM,IL,IG),
     *                                CC,AR)
C ***   RRJLT (IT)= RRJLT (IT)+CHISQR(D(IM,IL,IG),PJLT(IM,IL,IG),
C    *                                CC,AR)
**     PRODUCE MEAN ABSOLUTE DEVIATION MEASURE OF EFFECTIVENESS
        IF(D(IM,IL,IG) .NE. 0 .AND. PMLE(IM,IL,IG).NE.0
     *             .AND. PMLE(IM,IL,IG).NE.1) THEN
          CALL UOM( CC, AR, PMLE(IM,IL,IG), MOPMLE(IT),
     *              MUPMLE(IT), KVOPML(IT), KVUPML(IT), KVDPML(IT))
        ENDIF

        IF(CC.NE.0.) THEN
          CALL UOM( CC, AR, TSCA(IM,IL,IG), MOTSCA(IT),
     *              MUTSCA(IT), KVOTSA(IT), KVUTSA(IT), KVDTSA(IT))

          CALL UOM( CC, AR, TSCAM(IM,IL,IG), MOTSAM(IT),
     *              MUTSAM(IT), KVOTSM(IT), KVUTSM(IT), KVDTSM(IT))

          CALL UOM( CC, AR, PJ(IM,IL,IG), MOPJ(IT),
     *              MUPJ(IT), KVOPJ(IT), KVUPJ(IT), KVDPJ(IT))

          CALL UOM( CC, AR, PJM(IM,IL,IG), MOPJM(IT),
     *              MUPJM(IT), KVOPJM(IT), KVUPJM(IT), KVDPJM(IT))

          CALL UOM( CC, AR, PJ1(IM,IL,IG), MOPJ1(IT),
```

```
     *                MUPJ1(IT), KVOPJ1(IT), KVUPJ1(IT), KVDPJ1(IT))

           CALL UOM( CC, AR, PJ1M(IM,IL,IG), MOPJ1M(IT),
     *                MUPJ1M(IT), KVOP1M(IT), KVUP1M(IT), KVDP1M(IT))

           CALL UOM( CC, AR, PEB(IM,IL,IG), MOPEB(IT),
     *                MUPEB(IT), KVOPEB(IT), KVUPEB(IT), KVDPEB(IT))

           CALL UOM( CC, AR, PTEB(IM,IL,IG), MOPTEB(IT),
     *                MUPTEB(IT), KVOPTB(IT), KVUPTB(IT), KVDPTB(IT))

           CALL UOM( CC, AR, PEBM(IM,IL,IG), MOPEBM(IT),
     *                MUPEBM(IT), KVOPBM(IT), KVUPBM(IT), KVDPBM(IT))

           CALL UOM( CC, AR, PTEBM(IM,IL,IG), MOPTBM(IT),
     *                MUPTBM(IT), KVOBTM(IT), KVUBTM(IT), KVDBTM(IT))
          ENDIF

270      CONTINUE
260     CONTINUE
250    CONTINUE
       IF (KP .EQ. 0) THEN
           FF = 0
       ELSE
           FF=FLOAT(KV(IT))/FLOAT(KP)
       ENDIF
       RRML(IT)=RRML(IT)*FF
240   CONTINUE
**       COMPUTE TIME AVERAGE MAD
**       MEASURE OF EFFECTIVENESS
      DO 300 IT=1, NVYR
        MUPMLE(IT) = MUPMLE(IT)/KV(IT)
        MUTSCA(IT) = MUTSCA(IT)/KV(IT)
        MUTSAM(IT) = MUTSAM(IT)/KV(IT)
        MUPJ  (IT) = MUPJ (IT)/KV(IT)
        MUPJM (IT) = MUPJM(IT)/KV(IT)
        MUPJ1 (IT) = MUPJ1(IT)/KV(IT)
        MUPJ1M(IT) = MUPJ1M(IT)/KV(IT)
        MUPEB (IT) = MUPEB(IT)/KV(IT)
        MUPTEB (IT) = MUPTEB(IT)/KV(IT)
        MUPEBM(IT) = MUPEBM(IT)/KV(IT)
        MUPTBM(IT) = MUPTBM(IT)/KV(IT)
        MOPMLE(IT) = MOPMLE(IT)/KV(IT)
        MOTSCA(IT) = MOTSCA(IT)/KV(IT)
        MOTSAM(IT) = MOTSAM(IT)/KV(IT)
        MOPJ  (IT) = MOPJ (IT)/KV(IT)
        MOPJM (IT) = MOPJM(IT)/KV(IT)
        MOPJ1 (IT) = MOPJ1(IT)/KV(IT)
        MOPJ1M(IT) = MOPJ1M(IT)/KV(IT)
        MOPEB (IT) = MOPEB(IT)/KV(IT)
        MOPTEB (IT) = MOPTEB(IT)/KV(IT)
        MOPEBM(IT) = MOPEBM(IT)/KV(IT)
        MOPTBM(IT) = MOPTBM(IT)/KV(IT)
        MDPMLE(IT) = MUPMLE(IT) + MOPMLE(IT)
        MDTSCA(IT) = MUTSCA(IT) + MOTSCA(IT)
        MDTSAM(IT) = MUTSAM(IT) + MOTSAM(IT)
```

```
      MDPJ  (IT) = MUPJ  (IT) + MOPJ  (IT)
      MDPJM (IT) = MUPJM (IT) + MOPJM (IT)
      MDPJ1 (IT) = MUPJ1 (IT) + MOPJ1 (IT)
      MDPJ1M(IT) = MUPJ1M(IT) + MOPJ1M(IT)
      MDPEB (IT) = MUPEB (IT) + MOPEB (IT)
      MDPTEB (IT) = MUPTEB (IT) + MOPTEB (IT)
      MDPEBM(IT) = MUPEBM(IT) + MOPEBM(IT)
      MDPTBM(IT) = MUPTBM(IT) + MOPTBM(IT)
300   CONTINUE
************************************************************************
**       PRINT SECTION FOR ALL OUTPUT.                                **
************************************************************************
**       PRINT GROUP CHARACTERISTICS
      WRITE(6,'(1X,''TEN OR LESS BAYES VARIANCE ITERATIONS'')')
      WRITE(6,'(1X,I2,'' BAYES VARIANCE ITERATION(S)'')') IC
      WRITE(6,'(1X,''XBB '',F12.6)') XBB
      WRITE(6,905) T,KK,SSB,SSE,SHRX,SHR,ZB,ZTB,PZTB,A,AT
905   FORMAT(' GROUP CHARACTERISTICS', /,' T=',I6,  /,' K=',I6,
     *   /,'  SSB=',F12.4, /,'  SSE=',F12.4, /,' SHRX=',F12.4,
     *   /,'  SHR=',F12.4, /,'   ZB=',F12.4, /,'  ZTB=',F12.4,
     *   /,' PZTB=',F12.4, /,'    A=',F12.4, /,'   AT=',F12.4)
**       PRINT RESULTS FOR TRANSFORM SPACE
      WRITE(6,*) ' RESULTS FOR TRANSFORM SPACE'
      WRITE(6,906) (IVYR(I), I=1, NVYR)
906   FORMAT(14X,10I12)
      WRITE(6,907) 'RTSCA ', (RTSCA(IT), IT=1,NVYR)
907   FORMAT(' ',A14,10F12.4)
      WRITE(6,907) 'RJ    ', (RJ   (IT), IT=1,NVYR)
      WRITE(6,907) 'RJ1   ', (RJ1  (IT), IT=1,NVYR)
      WRITE(6,907) 'RXEB  ', (RXEB (IT), IT=1,NVYR)
      WRITE(6,907) 'RXTEB  ', (RXTEB (IT), IT=1,NVYR)
C *** WRITE(6,907) 'RJLT  ', (RJLT (IT), IT=1,NVYR)
      WRITE(6,907) 'RML   ', (RML  (IT), IT=1,NVYR)
C *** WRITE(6,907) 'RSL   ', (RSL  (IT), IT=1,NVYR)
C *** WRITE(6,907) 'RTJ   ', (RTJ  (IT), IT=1,NVYR)
C *** WRITE(6,907) 'RTJLT ', (RTJLT(IT), IT=1,NVYR)
**       PRINT RESULTS FOR ORIGINAL SPACE
      WRITE(6,*)    ' '
      WRITE(6,*) ' RESULTS FOR ORIGINAL SPACE'
      WRITE(6,907) 'RRML  ', (RRML (IT), IT=1,NVYR)
      WRITE(6,907) 'RRTSCA', (RRTSCA(IT), IT=1,NVYR)
      WRITE(6,907) 'RRTSAM', (RRTSAM(IT), IT=1,NVYR)
      WRITE(6,907) 'RRJ   ', (RRJ  (IT), IT=1,NVYR)
      WRITE(6,907) 'RRJM  ', (RRJM (IT), IT=1,NVYR)
      WRITE(6,907) 'RRJ1  ', (RRJ1 (IT), IT=1,NVYR)
      WRITE(6,907) 'RRJ1M ', (RRJ1M (IT), IT=1,NVYR)
      WRITE(6,907) 'RRPEB ', (RRPEB(IT), IT=1,NVYR)
      WRITE(6,907) 'RRPTEB ', (RRPTEB(IT), IT=1,NVYR)
      WRITE(6,907) 'RRPEBM', (RRPEBM(IT), IT=1,NVYR)
      WRITE(6,907) 'RRPTBM', (RRPTBM(IT), IT=1,NVYR)
C *** WRITE(6,907) 'RRJLT ', (RRJLT(IT), IT=1,NVYR)
      WRITE(6,908) 'KV    ', (KV   (IT), IT=1,NVYR)
908   FORMAT(' ',A14,10I12)
      WRITE(6,'(9X,''KP    '',I12)') KP
      WRITE(6,*)    ' '
**       PRINT MAD CALCULATIONS
```

```
      WRITE(6,'(2X,''PMLE'')')
      CALL WRMAD(IVYR,NVYR,MUPMLE,KVUPML,MOPMLE,KVOPML,MDPMLE,KVDPML)
      WRITE(6,'(2X,''TSCA'')')
      CALL WRMAD(IVYR,NVYR,MUTSCA,KVUTSA,MOTSCA,KVOTSA,MDTSCA,KVDTSA)
      WRITE(6,'(2X,''TSCAM'')')
      CALL WRMAD(IVYR,NVYR,MUTSAM,KVUTSM,MOTSAM,KVOTSM,MDTSAM,KVDTSM)
      WRITE(6,'(2X,''PJ'')')
      CALL WRMAD(IVYR,NVYR,MUPJ,KVUPJ,MOPJ,KVOPJ,MDPJ,KVDPJ)
      WRITE(6,'(2X,''PJM'')')
      CALL WRMAD(IVYR,NVYR,MUPJM,KVUPJM,MOPJM,KVOPJM,MDPJM,KVDPJM)
      WRITE(6,'(2X,''PJ1'')')
      CALL WRMAD(IVYR,NVYR,MUPJ1,KVUPJ1,MOPJ1,KVOPJ1,MDPJ1,KVDPJ1)
      WRITE(6,'(2X,''PJ1M'')')
      CALL WRMAD(IVYR,NVYR,MUPJ1M,KVUP1M,MOPJ1M,KVOP1M,MDPJ1M,KVDP1M)
      WRITE(6,'(2X,''PEB'')')
      CALL WRMAD(IVYR,NVYR,MUPEB,KVUPEB,MOPEB,KVOPEB,MDPEB,KVDPEB)
      WRITE(6,'(2X,''PEBM'')')
      CALL WRMAD(IVYR,NVYR,MUPEBM,KVUPBM,MOPEBM,KVOPBM,MDPEBM,KVDPBM)
      WRITE(6,'(2X,''PTEB'')')
      CALL WRMAD(IVYR,NVYR,MUPTEB,KVUPTB,MOPTEB,KVOPTB,MDPTEB,KVDPTB)
      WRITE(6,'(2X,''PTEBM'')')
      CALL WRMAD(IVYR,NVYR,MUPTBM,KVUBTM,MOPTBM,KVOBTM,MDPTBM,KVDBTM)
      WRITE(6,*)  ' '
**       PRINT THE CENTRAL INVENTORY MATRIX
      WRITE(6,*) 'CINV MATRIX'
      DO 1006 IT=1,NYR
       WRITE(6,*) 'YEAR',IT
       DO 1005 IL=1,NLOS
        WRITE(6,1007)(CINV(IM,IL,1,IT),IM=1,NMOS)
1005   CONTINUE
1006  CONTINUE
1007  FORMAT(15(1X,F7.2))
      WRITE(6,*)  ' '
**       PRINT THE ATTRITION MATRIX
      WRITE(6,*) 'Y MATRIX'
      DO 1016 IT=1,NYR
       WRITE(6,*) 'YEAR',IT
       DO 1015 IL=1,NLOS
        WRITE(6,1007)(Y(IM,IL,1,IT),IM=1,NMOS)
1015   CONTINUE
1016  CONTINUE
      WRITE(6,*)  ' '
**       PRINT ALL ARRAYS
      WRITE(6,'(2X,''ARRAY TSCA'')')
      CALL ARRAY(MXMOS,MXLOS,MXGRD,NMOS,NLOS,NGRD,TSCA)
      WRITE(6,'(2X,''ARRAY TSCAM'')')
      CALL ARRAY(MXMOS,MXLOS,MXGRD,NMOS,NLOS,NGRD,TSCAM)
      WRITE(6,'(2X,''ARRAY SHREB'')')
      CALL ARRAY(MXMOS,MXLOS,MXGRD,NMOS,NLOS,NGRD,SHREB)
      WRITE(6,'(2X,''ARRAY SHRTEB'')')
      CALL ARRAY(MXMOS,MXLOS,MXGRD,NMOS,NLOS,NGRD,SHRTEB)
      WRITE(6,'(2X,''ARRAY PMLE'')')
      CALL ARRAY(MXMOS,MXLOS,MXGRD,NMOS,NLOS,NGRD,PMLE)
      WRITE(6,'(2X,''ARRAY PJ'')')
      CALL ARRAY(MXMOS,MXLOS,MXGRD,NMOS,NLOS,NGRD,PJ)
      WRITE(6,'(2X,''ARRAY PJM'')')
```

```
      CALL ARRAY(MXMOS,MXLOS,MXGRD,NMOS,NLOS,NGRD,PJM)
      WRITE(6,'(2X,''ARRAY PJ1'')')
      CALL ARRAY(MXMOS,MXLOS,MXGRD,NMOS,NLOS,NGRD,PJ1)
      WRITE(6,'(2X,''ARRAY PJ1M'')')
      CALL ARRAY(MXMOS,MXLOS,MXGRD,NMOS,NLOS,NGRD,PJ1M)
      WRITE(6,'(2X,''ARRAY PEB'')')
      CALL ARRAY(MXMOS,MXLOS,MXGRD,NMOS,NLOS,NGRD,PEB)
      WRITE(6,'(2X,''ARRAY PTEB'')')
      CALL ARRAY(MXMOS,MXLOS,MXGRD,NMOS,NLOS,NGRD,PTEB)
      WRITE(6,'(2X,''ARRAY PEBM'')')
      CALL ARRAY(MXMOS,MXLOS,MXGRD,NMOS,NLOS,NGRD,PEBM)
      WRITE(6,'(2X,''ARRAY PTEBM'')')
      CALL ARRAY(MXMOS,MXLOS,MXGRD,NMOS,NLOS,NGRD,PTEBM)
      WRITE(6,'(2X,''ARRAY V'')')
      CALL ARRAY(MXMOS,MXLOS,MXGRD,NMOS,NLOS,NGRD,V)
      WRITE(6,'(2X,''ARRAY XB'')')
      CALL ARRAY(MXMOS,MXLOS,MXGRD,NMOS,NLOS,NGRD,XB)
      WRITE(6,'(2X,''ARRAY VT'')')
      CALL ARRAY(MXMOS,MXLOS,MXGRD,NMOS,NLOS,NGRD,VT)
      WRITE(6,'(2X,''ARRAY XTB'')')
      CALL ARRAY(MXMOS,MXLOS,MXGRD,NMOS,NLOS,NGRD,XTB)
      WRITE(6,'(2X,''ARRAY XJ'')')
      CALL ARRAY(MXMOS,MXLOS,MXGRD,NMOS,NLOS,NGRD,XJ)
      WRITE(6,'(2X,''ARRAY XJ1'')')
      CALL ARRAY(MXMOS,MXLOS,MXGRD,NMOS,NLOS,NGRD,XJ1)
      WRITE(6,'(2X,''ARRAY XEB'')')
      CALL ARRAY(MXMOS,MXLOS,MXGRD,NMOS,NLOS,NGRD,XEB)
      WRITE(6,'(2X,''ARRAY XTEB'')')
      CALL ARRAY(MXMOS,MXLOS,MXGRD,NMOS,NLOS,NGRD,XTEB)
*
      END
**********************************************************************
      SUBROUTINE WRMAD(IYR,NYR,MU,KU,MO,KO,MD,KD)
**********************************************************************
      INTEGER IYR(NYR),KU(NYR),KO(NYR),KD(NYR)
      REAL    MU(NYR),MO(NYR),MD(NYR)
      WRITE(6,'(/9X,''DEVIATION'',T26,I2,T42,I2,T59,I2/)')
     *        (IYR(IW),IW=1,NYR)
      WRITE(6,'(9X,''UNDERAGE'',T22,F9.6,'' ('',I2,'')'',T38,F9.6,
     *         '' ('',I2,'')'',T55,F9.6,'' ('',I2,'')'')')
     *         MU(1),KU(1),MU(2),KU(2),MU(3),KU(3)
      WRITE(6,'(9X,''OVERAGE'',T22,F9.6,'' ('',I2,'')'',T38,F9.6,
     *         '' ('',I2,'')'',T55,F9.6,'' ('',I2,'')'')')
     *         MO(1),KO(1),MO(2),KO(2),MO(3),KO(3)
      WRITE(6,'(9X,''TOTAL'',T22,F9.6,'' ('',I2,'')'',T38,F9.6,
     *         '' ('',I2,'')'',T55,F9.6,'' ('',I2,'')'')')
     *         MD(1),KD(1),MD(2),KD(2),MD(3),KD(3)
      WRITE(6,'(/)')
      RETURN
      END
**********************************************************************
      SUBROUTINE ARRAY(MM,ML,MG,NM,NL,NG,OBJ1)
**********************************************************************
      REAL OBJ1(MM,ML,MG)
      DO 10 IG1=1,NG
```

```
      DO 10 IL1=1,NL
10       WRITE(6,'(2X,15(F7.3,1X))') (OBJ1(IM1,IL1,IG1),IM1=1,NM)
      WRITE(6,'(/)')
      RETURN
      END
*********************************************************************
      SUBROUTINE FTT(CINV,Y, X)
*********************************************************************
      REAL CINV,X,Y
      TEMP=Y/(1+CINV)
      TEMP1=(1.+Y)/(1+CINV)
      TEMP=SQRT(.5+CINV)*.5*(ASIN(-1.+2.*TEMP) +
     *                    ASIN(-1.+2.*TEMP1))
      X=TEMP
      END
*********************************************************************
      SUBROUTINE UOM(VINV,ACT,OBJ,OVER,UNDER,KO,KU,KD)
*********************************************************************
      REAL OBJ
      IF (ACT .GT. OBJ) THEN
        UNDER = UNDER+VINV*(ACT-OBJ)
        KU=KU+1
      ELSE
        OVER  = OVER +VINV*(OBJ-ACT)
        KO=KO+1
      ENDIF
      KD=KD+1
      RETURN
      END
*********************************************************************
      REAL FUNCTION RHO(XB,XBB,SSE,SSB,DEE,KK,T,NN)
*********************************************************************
      INTEGER T
      REAL*8 SIGSQ, V, U
      SIGSQ=SSE/(NN+2)
      V=SSB/T
      U=KK*(KK-3)*(XB-XBB)**2/(V*(KK-1))
      RHO=DMIN1(1.D0, DEE/DSQRT(U))
      END
*********************************************************************
      REAL FUNCTION MILLER( X, AVGINV )
*********************************************************************
      TM = X/SQRT(AVGINV+.5)+1.570796
      TEMP1 = ASIN(SQRT(1./(AVGINV+1)))
      IF (TM .LT. TEMP1) THEN
        MILLER = 0.0
        GOTO 1
      ENDIF
      IF (TM .GT. 3.14159-TEMP1) THEN
        MILLER = 1.0
        GOTO 1
      ENDIF
      IF (COS(TM) .EQ. 0.0) THEN
        MILLER = 0.5
        GOTO 1
      ENDIF
```

```
      IF (COS(TM) .GT. 0.0) SIGNUM = 1.
      IF (COS(TM) .LT. 0.0) SIGNUM = -1.
      TEMP2=1-(SIN(TM)+(SIN(TM)-(1./SIN(TM)))/AVGINV)**2
      IF (TEMP2.LT.0.0) TEMP2=0.0
      MILLER=.5*(1.-SIGNUM*TEMP2**.5)
1     END
**********************************************************************
      REAL FUNCTION SCINV(X,A)
**********************************************************************
**          SCALE INVERSION (TRANSFORM ==> ORIGINAL SCALE)
      R=X/SQRT(A+.5)
      IF(R .LT. -1.570796) THEN
          SCINV=0.
      ELSE IF(R .GT.  1.570796) THEN
          SCINV=1.
      ELSE
          SCINV=.5*(1.+SIN(R))
      ENDIF
      END
**********************************************************************
      REAL FUNCTION VAR( ZZ )
**********************************************************************
      DATA AA/1.6835/, B1/-.8934/, B2/.8991/
      IF ( ZZ .GT. 2.2) THEN
        VAR = 1.
      ELSE
        IF ( ZZ .LT. 1.001) THEN
          VAR = .05
        ELSE
          VAR = AA*(ZZ**B1)*(ZZ-1.)**B2
        ENDIF
      ENDIF
      END
**********************************************************************
      REAL FUNCTION CHISQR(ID,OBJ,INV,ACT)
**********************************************************************
      REAL OBJ,INV
      IF (ID.NE.0 .AND. OBJ.NE.0. .AND. OBJ.NE.1.)
     *  CHISQR = (INV*(ACT-OBJ)**2)/(OBJ*(1.-OBJ))
      END
**********************************************************************
      SUBROUTINE READER(MOS,LOS,GRADE,YR,INV,Y, NMOS,SMOS,
     *   NGRD,SGRD, SLOS1,SLOS2, ST1,ST2, IM,IL,IG,IT, IEOF)
**********************************************************************
**          READ RECORD AND ACCUMULATE LOSSES
      INTEGER MOS, LOS, GRADE, YR, INV,  LOSS(8)
      REAL Y
      INTEGER ST1, ST2
      INTEGER SLOS1, SLOS2
      INTEGER SMOS(40), NMOS
      INTEGER SGRD(10), NGRD
      ICNT=0
1     READ(1,100,END=999) YR,MOS,GRADE,LOS,INV,(LOSS(I),I=1,8)
100   FORMAT(4I2,9I5)
**          CHECK IF RECORD MEETS SELECTION CRITERIA.  OTHERWISE REJECT.
      IM=0
```

```
      DO 10 I=1,NMOS
       IF(MOS .EQ. SMOS(I)) THEN
              IM=I
              GO TO 20
       END IF
10    CONTINUE
      GO TO 1
*
20    CONTINUE
      DO 30 I=1,NGRD
       IF(GRADE .EQ. SGRD(I)) THEN
              IG=I
              GO TO 40
       END IF
30    CONTINUE
      GO TO 1
40    CONTINUE
*
      IF(LOS .LT. SLOS1 .OR. LOS .GT. SLOS2) GO TO 1
      IL=LOS-SLOS1+1
**       YEARS OVER ST2 ARE USED FOR VALIDATION
      IF(YR .LT. ST1) GO TO 1
      IT=YR-ST1+1
**       COMPUTE TOTAL LOSS
      Y=0
      DO 50 I=1,8
       Y=Y + LOSS(I)
50    CONTINUE
*
      RETURN
999   IEOF=1
      END
```

APPENDIX C. SIMULATION FOR CHOICE OF AVERAGE INVENTORY VALUES

```
      PROGRAM MCSIM
**********************************************************************
*     SIMULATION COMPARISON OF INVENTORY MEAN PERFORMANCE            *
*                                                                    *
*     PURPOSE: TO IDENTIFY THE BEST MEAN VALUE (ARITHMETIC,          *
*              GEOMETRIC AND HARMONIC) FOR USE IN THE INVERSION OF   *
*              TRANSFORMED DATA TO THE ORIGINAL SPACE.               *
*     PROGRAMMER: CAPT C R DICKINSON, USMC                           *
*     VARIABLE DESCRIPTION:                                          *
*              LAMBDA - PARAMETER USED IN THE POISSON RANDOM NUMBER  *
*                       GENERATOR                                    *
*              TRIALS - VECTOR OF INVENTORY VALUES GENERATED FROM    *
*                       THE POISSON RANDOM NUMBER GENERATOR          *
*              REPS   - NUMBER OF INVENTORY VALUES TO BE GENERATED   *
*              BASIC  - BASIC INVERSION OF TRANSFORMED VALUES        *
*              FTE1   - FREEMAN-TUKEY EXACT INVERSION WITH           *
*                                                                    *
*              FTE2   - FREEMAN-TUKEY EXACT INVERSION WITH           *
*                                                                    *
*              Z      - TRANSFORMED SCALE VARIABLE                   *
*              MEANS  - VECTOR OF ARITHMETIC, GEOMETRIC AND HARMONIC *
*                       MEANS                                        *
*              TRSUM  - SUMMATION OF TRIALS FOR USE IN THE           *
*                       ARITHMETIC MEAN                              *
*              TRMULT - MULTIPLICATION OF TRIALS FOR USE IN THE      *
*                       GEOMETRIC MEAN                               *
*              TRINV  - SUMMATION OF THE INVERSE OF TRIALS FOR USE   *
*                       IN THE HARMONIC MEAN                         *
*     INPUT/OUTPUT:                                                  *
*              INPUT - HARD CODED PARAMETERS                         *
*             OUTPUT - SIM OUT A1 (DISK FILE 03)                     *
**********************************************************************
*
***      VARIABLE DECLARATION
      INTEGER TRIALS(100),Y(100),REPS(3),PROB,R,
     C     COUNT,TRSUM,YTEMP,YSUM
      REAL Z(100),T(100),MEAN(3),BASIC(3),DIFF(3),LAMBDA(5),
     C     P(4),FTE1(3),FTE2(3),TRMULT
      DOUBLE PRECISION DPSEED
*
      CALL EXCMS('FILEDEF 02 DISK REPLIC LISTING A1')
*
***      VARIABLE INITIALIZATION
      PI=3.14159
      DPSEED=889246.D0
      LAM=5
      R=3
      PROB=4
      COUNT=2
```

```
      DATA REPS/20,40,60/
      DATA LAMBDA/1.,2.,4.,8.,16./
      DATA P/.05,.1,.2,.4/
*
***      HEADER PRINT
      WRITE (02,'(/T6,''LAMBDA'',T15,''REPS'',T27,''P'',
     C      T34,''METHOD'',T46,''ARITHMETIC'',T59,
     C      ''GEOMETRIC'',T71,''HARMONIC''/)')
*
***      LOOP FOR LAMBDA PARAMETER
      DO 500 I=1,LAM
         DO 450 J=1,R
            CALL GGPOS(LAMBDA(I),DPSEED,REPS(J),TRIALS,IER)

            NSUM=0
            DO 10 J1=1,REPS(J)
               TRIALS(J1)=TRIALS(J1)+1
10             NSUM=NSUM+TRIALS(J1)
*
***      LOOP FOR PROBABILITY PARAMETER
            DO 400 K=1,PROB
              COUNT=COUNT+4
***      LINE COUNTER FOR PAGE BREAK
              IF (COUNT.GT.55) THEN
                 WRITE (2,'(1H1)')
                 COUNT=2
                 WRITE (02,'(/T6,''LAMBDA'',T15,''REPS'',T27,''P'',
     C                 T34,''METHOD'',T46,''ARITHMETIC'',T59,
     C                 ''GEOMETRIC'',T71,''HARMONIC''/)')
              ENDIF
*
               TRSUM=0
               TRINV=0
               TRMULT=0.
               ARCSUM=0.
               TSUM=0.
               YSUM=0
*
***      LOOP FOR REPITITION PARAMETER
               DO 20 K1=1,REPS(J)
                  CALL GGBN(DPSEED,1,TRIALS(K1),P(K),YTEMP)
                  YSUM=YSUM+YTEMP
                  TRSUM=TRSUM+TRIALS(K1)
                  TRMULT=TRMULT+ALOG(REAL(TRIALS(K1)))
                  TRINV=TRINV+1./REAL(TRIALS(K1))
                  Y(K1)=YTEMP
                  Z(K1)=.5*(TRIALS(K1)+.5)**.5*(ASIN(2.*Y(K1)/
     C                 (TRIALS(K1)+1.)-1.)+ASIN(2.*(Y(K1)+1.)/
     C                 (TRIALS(K1)+1.)-1.)+PI)
                  T(K1)=Z(K1)/((TRIALS(K1)+.5)**.5)
                  ARCSUM=ARCSUM+Z(K1)
20                TSUM=TSUM+T(K1)
*
               ZAVG=ARCSUM/REAL(REPS(J))
               TAVG=TSUM/REAL(REPS(J))
***      MEAN VALUE CALCULATION
```

```
            MEAN(1)=TRSUM/REAL(REPS(J))
            MEAN(2)=EXP(TRMULT*(1./REAL(REPS(J))))
            MEAN(3)=REAL(REPS(J))/TRINV
*
            IF (COS(TAVG).GE.0.0) THEN
               SIGNUM=1.
            ELSE
               SIGNUM=-1.
            ENDIF
*
            DO 30 K2=1,3
               TAVG1=ZAVG/(MEAN(K2)+.5)**.5
               IF (COS(TAVG1).GE.0.0) THEN
                  SIG1=1.
               ELSE
                  SIG1=-1.
               ENDIF
***      BASIC AND FTE INVERSE METHOD CALCULATION
               FTE1(K2)=.5*(1.-(SIGNUM*(1.-(ABS(SIN(TAVG)+
     C            (SIN(TAVG)-(1./SIN(TAVG)))/MEAN(K2)))**2.)**.5))
*
               FTE2(K2)=.5*(1.-(SIG1*(1.-(ABS(SIN(TAVG1)+
     C            (SIN(TAVG1)-(1./SIN(TAVG1)))/MEAN(K2)))**2.)**.5))
*
30             BASIC(K2)=.5*(1.+SIN(ZAVG/((MEAN(K2)+.5)**.5)
     C            -(PI/2.)))
***      OUTPUT STATEMENTS
         DO 40 K3=1,3
            IF (K3.EQ.1) THEN
               WRITE (02,'(T6,F4.1,T15,I3,T25,F5.3,T35,''BASIC'',
     C               T46,F7.4,T59,F7.4,T71,F7.4)')
     C               LAMBDA(I),REPS(J),P(K),(BASIC(IK),IK=1,3)
            ELSE
               IF (K3.EQ.2) THEN
                  WRITE (02,'(T35,''FTE1'',T46,F7.4,T59,F7.4,
     C                  T71,F7.4)') (FTE1(IK),IK=1,3)
               ELSE
                  WRITE (02,'(T35,''FTE2'',T46,F7.4,T59,F7.4,
     C                  T71,F7.4/)') (FTE2(IK),IK=1,3)
               ENDIF
            ENDIF
40       CONTINUE
*
400      CONTINUE
450   CONTINUE
500 CONTINUE
    STOP
    END
```

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